PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Distribution of the Magnetic Field and Return Current Round a Submarine Cable Carrying Alternating Current. Part 1

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III.—*The Distribution of the Magnetic Field and Return Current round a Submarine Cable carrying Alternating Current.—Part* 1.

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Synopsis.

I. *Introductory.*—Importance of knowledge of field round cable for leader gear, and of propagation of electromagnetic impulses in the sea in connection with signalling. Brief history and outline of the research.

II. *Theory.*—Propagation of plane electromagnetic waves in a conducting isotropic medium. Table of theoretical velocities, wave-lengths, and attenuation for various frequencies in sea water. Theory of cylindrical conductors. Principle of similitude, enabling field above water to be determined by a model employing a metal sheet in place of the water. Mr. Butterworth's theory for thin conducting lamina.

III. *Preliminary Experiments.*—Measurements of absorption at low frequencies at Harwich in 1918. Determination of resistivity of sea water. Leader Gear. Preliminary tests of field distribution. Visual Leader Gear. Tests with inclined coils on steel ship. Experiments of H. Lichte at Kiel.

IV. *Systematic Experimental Research.*—Arrangement of cables and testing barge at the Admiralty Experimental Station, Shandon. Alternating current potentiometer method of determining intensity and phase of magnetic field, and of the current density in the water. Results at frequencies of 15, 50, 120, 250 and 500 periods per second. Effect of depth of water. Isorheal and Isophasal lines. Effect of cable armouring. Comparison of results with theory, and conclusions.

I. Introductory.

Among the scientific problems to which the late war gave rise, one of the greatest fundamental and practical importance is the determination of the conditions of propagation of impulses or disturbances of every kind below and above the surface of the sea. As sea water is a medium of fairly high electrical conductivity, the propagation of electromagnetic waves, especially of high frequencies, is greatly affected by the absorption in the water and the distortion at the surface. In view of the applications to signalling, the guiding of ships over submarine cables, and the actuation of mechanisms, a knowledge of the distribution of the intensity and direction of the magnetic field produced by a submarine cable or loop is of the greatest practical importance, apart from its scientific interest.

The investigations described in the present paper commenced in January, 1918, when the writer was called to the Admiralty Experimental Station at Parkeston Quay, Harwich, for the purpose of devising control mechanisms actuated by submarine cables. It was immediately obvious that absorption by the sea water might prove to be a serious factor in diminishing the intensity of the magnetic field, and that low frequencies would therefore be preferable. Calculations were therefore made concerning the propagation of electromagnetic waves in the sea, but as these applied only to plane waves in an infinite medium it was also decided to make a few direct measurements on the absorption of the field produced by a 300 by 200 yard loop which had been laid down at the mouth of Harwich harbour, the result being to show that absorption had no serious effect at frequencies of 10 to 20 $\sim$ per second. This result was sufficient for the immediate purpose.

About June, 1918, experiments were commenced on the method of guiding ships along cables now known as Leader Gear, and it became desirable to obtain further information concerning the distribution of the magnetic field around a submerged cable. The writer therefore made proposals for a complete investigation, employing search coils in conjunction with an alternating current potentiometer, which would enable the magnitude and phase of the magnetic field to be determined.

Preliminary experiments establishing the validity of this method were made by Mr. Rivers Moore at Parkeston Quay, but the exigencies of war conditions did not permit of an exhaustive research. After the Armistice the whole of the experimental work was removed to the Shandon Experimental Station on the Gareloch, which offered exceptionally favourable conditions for this research owing to the depth and sheltered nature of the Loch, and the writer then put forward a scheme for an accurate investigation of the field distribution both above and below the surface of the water, and of the distribution of the return current in the water.

For this purpose it was proposed to employ a wooden barge of shallow draught and fitted with a sliding vertical mast carrying two pairs of horizontal and vertical search coils at the top and bottom, thus enabling the horizontal and vertical components of the magnetic field to be determined both above and below the surface of the water and at various heights and depths. To enable the density of the return current in the water to be measured it was proposed to employ a horizontal spar 10 metres long fixed to the bottom of the mast with silver electrodes at its ends, from which leads were to be brought along to the centre of the spar and connected by a twin cable to the potentiometer. This arrangement eliminates all inductive effects and enables the density and phase of the current in the water to be determined if its resistivity is known.

Although facilities for carrying out the research on the scale originally proposed, involving a barge carrying a 100-foot mast, so that the field and current distribution could be determined from the bottom to 100 feet above the surface were not granted, preparations were made in 1919 for a research on a smaller scale employing an existing barge, the "St. Adrian," with a 40-foot mast carrying four coils each of about 1 square metre in area, and having an effective total area of 400 square metres. Two cables, each about 3 miles of twin core armoured cable, were laid down in the Gareloch for these and other researches.

The preparations for this research were very lengthy, and measurements were not actually started till June, 1920. In the meantime, however, the importance of a knowledge of the magnetic field distribution above the water was so strongly felt in connection with a scheme for leader gear employing visual indicators that approximate determinations were made on a steel ship, H.M.S. "Auricula," which had been fitted with two large inclined coils for the leader gear experiments.

Before these tests were made it had been assumed that as the amount of absorption found in previous experiments at frequencies of 10 to 20 ~ per second was small, the distortion of the field from the ordinary circular distribution would also be small, and that therefore by employing inclined coils in the rigging of the ship, equal E.M.F.'s would be induced when the ship was vertically over the cable, and that the ratio of the E.M.F.'s would vary with the lateral range, allowing the steering to be accomplished by the operation of alternating current relays and a lamp indicator. Although this device worked excellently as regards sensitiveness, ambiguous steering indications were obtained, and the measurements of the E.M.F.'s developed at different ranges, determined by a vibration of galvanometer showed that even at the low frequency of 15 ~ per second employed the distortion of the field was very serious, the lines of force apparently becoming horizontal at a lateral range of about 80 yards from the cable in a depth of about 16 fathoms. By reducing the sensitiveness of the relays so as to restrict the range of operation to a narrow belt, and carefully pairing the relays, satisfactory steering indications were obtained, but the need for a thorough investigation of the field distribution was emphasised. Incidentally these measurements indicated that at some part of the range the field was of an elliptically rotating character, as was anticipated from theoretical considerations.

By the end of 1920 determinations of the field and current distribution round one of the cables had been completed for frequencies of 15 ~ and 50 ~ per second, but the decision to transfer the station to Teddington threatened to prevent the extension of the investigation to higher frequencies.

By the kindness of the Physics Board of the Department of Scientific and Industrial Research, who gave a grant of £4000 for the continuance of the research, a small sub-station was erected on the shore of the Gareloch close to the original station in charge of Mr. S. J. Willis and Mr. L. Champney, B.Sc., a generating plant, battery and the necessary alternators were installed, and a new barge was obtained and equipped in place of the "St. Adrian," which had been totally wrecked in a storm on December 3rd. The equipment of the new sub-station took a considerable time, but experiments were resumed in September, 1921, and were carried on till the final closing down in April, 1922. During this period the field and current distribution was determined for frequencies of 120 ~, 250, and 500 ~ per second; the effect of variation of depth on the intensity of the field was investigated, and the effect of the armouring of the cable in reducing the field by concentration of the return current in the armouring was measured.

At the same time the mathematical theory of the subject has been considerably developed by Mr. S. BUTTERWORTH, M.SC.,* who has completely solved the problem of the magnetic field distribution in the air above the surface, and has obtained results in close agreement with the experimental measurements. He has also made use of the principle of similitude suggested by the writer to verify his theory by laboratory measurements employing a lead sheet in place of water.

As a result of all these investigations, the problem of the propagation of electromagnetic impulses in the sea may be said to have been completely solved, as the agreement of the actual sea measurements with the theory at frequencies up to 500 ~ per second is sufficiently close as to justify the theory and warrant its application to higher frequencies.

Certain investigations with a similar object have been made by other observers, notably by Herr H. LICHTE, of Kiel, and these will be referred to in their proper place; but it is believed that the research here described is the only one in which complete quantitative information concerning the magnitude, phase and direction of the magnetic field and current distribution has been obtained and verified by theory.

A large amount of experimental work on the practical application and development of Leader Gear has been carried out by H.M. Signal School, Portsmouth, during which the range of audible signals, the effect of depth, the screening effect of steel hulls, and the attenuation in the cable has been measured. These investigations are not dealt with in the present communication, but from a few comparisons which have been made there appears to be fair agreement with the conclusions derived from this research.

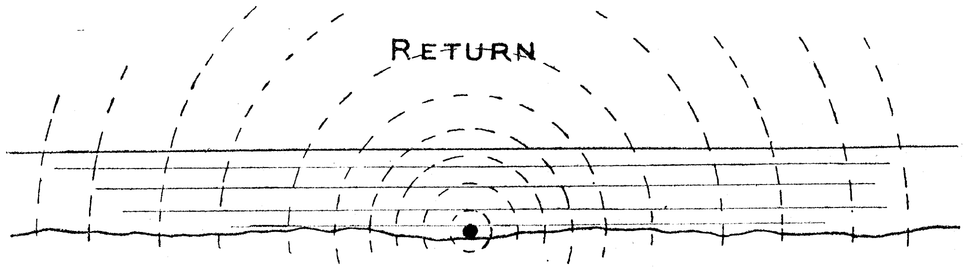
II. THEORY.

A rough quantitative theory of the distribution of the return current and magnetic field was propounded when the investigations were first commenced in 1918. Up to that time it appeared to have been generally assumed that the return current would tend to concentrate near the surface of the water, by a mistaken analogy to the skin effect in conductors. It was obvious, however, that this should only apply to currents flowing through the sea between electrodes, and that the skin effect should be in the opposite direction when the sea was used as a return to an insulated cable, as being comparable with a concentric return, so that the return current should tend to concentrate in on the cable and away from the surface. Owing, however, to the relatively low conductivity of the sea bottom beneath the cable, it was evident that this con-

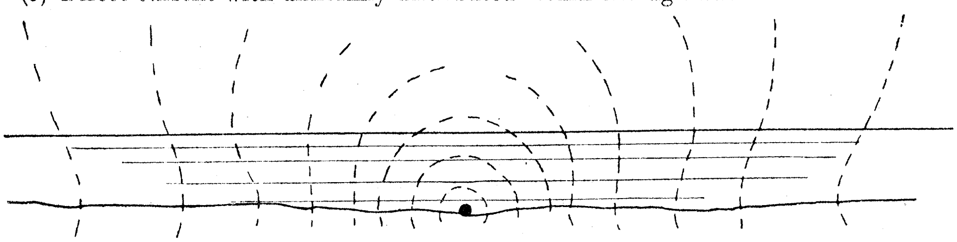
* See paper following.

centration could only take place above the cable, and that the magnetic field would, therefore, be of the form produced by a linear conductor with a distributed return parallel to and above it. This led to the expectation that the magnetic field would be of the type shown in fig. 1*c* instead of the ordinary circular distribution due to a single current (fig. 1*a*).

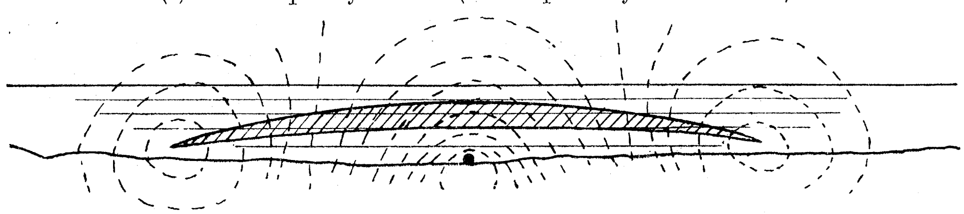
(*a*) Direct current (infinitely distant return).



(*b*) Direct current with uniformly distributed return through water of finite width.



(*c*) Low-frequency current (return partially concentrated).



(*d*) High-frequency current (concentrated return).

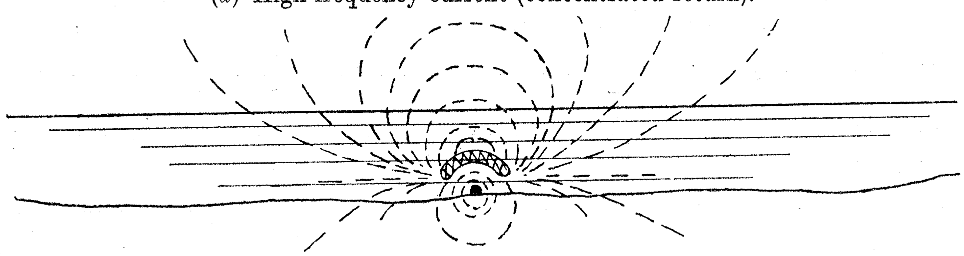


Fig. 1—Approximate Theoretical Field Distribution.

It had already been observed by Dr. F. B. Young, at Parkeston Quay in 1917, that the magnetic field at a considerable distance from a cable carrying alternating current of sonic frequency was horizontal instead of vertical, and a rough verification of the above suggested distribution was made by Messrs. J. T. Irwin and H. H. Rivers Moore at Parkeston Quay, at the writer's suggestion in June, 1918, employing a tilting coil and telephone receivers to determine the direction of the field, indicating a field

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distribution resembling fig. 1*d*. For the purpose immediately in view the writer was chiefly concerned at that time with the absorption of the field by the water, and the theory of the propagation of plane electromagnetic waves in an infinite conducting medium was worked out of the ordinary Maxwellian equations as follows :—

Propagation of Electromagnetic Waves in a Conducting Isotropic Medium.

If Δ is the current density in the medium, D the displacement, E the intensity of the electric field, H and B the magnetic force and induction, K and μ the dielectric constant and permeability of the medium, ρ its specific resistance, and u the ratio of the units (3×10^{10} cm./sec.), we have in C.G.S. electromagnetic measure—

$$\text{(i)}\quad \Delta = \dot{D} = \frac{1}{\rho}E + \frac{K}{4\pi u^2}\dot{E}, \qquad \text{(ii)}\quad B = \mu H,$$

$$\text{(iii)}\quad \textit{Curl}\ E = \dot{B}, \qquad \text{(iv)}\quad \textit{Curl}\ H = 4\pi\Delta = 4\pi\dot{D},$$

which gives the relation $\nabla^2 E - \frac{4\pi\mu}{\rho}\dot{E} - \frac{K\mu}{u^2}\ddot{E} = 0$, since $\nabla E = 0$; and similarly for H; so that if x is the distance in the direction of propagation,

$$\frac{\partial^2 y}{\partial x^2} - \frac{4\pi\mu}{\rho}\frac{\partial y}{\partial t} - \frac{K\mu}{u^2}\frac{\partial^2 y}{\partial t^2} = 0,$$

where y is either E or H.

Assuming $y = Ae^{-ax}\sin(\omega t - bx)$ where $\omega = 2\pi f$ and f is the frequency, then the velocity of propagation $V = \omega/b$ and the wave-length $\lambda = 2\pi/b$ and we have

$$a^2 - b^2 + \frac{K\mu\omega^2}{u^2} = 0 \qquad \text{and} \qquad ab = \frac{2\pi\mu\omega}{\rho},$$

from which

$$a = \left[\left\{\left(\frac{K\mu\omega^2}{2u^2}\right)^2 + \left(\frac{2\pi\mu\omega}{\rho}\right)^2\right\}^{\frac{1}{2}} - \frac{K\mu\omega^2}{2u^2}\right]^{\frac{1}{2}} \qquad \text{and} \qquad b = \frac{2\pi\mu\omega}{\rho a}.$$

Case I.—If ρ or ω is large, so that the second term is negligible in comparison with the first :— $a = \frac{2\pi u}{\rho}\sqrt{\frac{\mu}{K}}$ which is independent of frequency, and $b = \frac{\omega\sqrt{K\mu}}{u}$ approximately, so that $V = \frac{u}{\sqrt{K\mu}}$ and $\lambda = \frac{u}{f\sqrt{K\mu}}$. The velocity and wave-length are therefore independent of the resistivity, but the attenuation is proportional to the conductivity and is zero for a perfect insulator.

Case II.—If ρ or ω is small, so that $2\pi\mu\omega/\rho$ is large compared with $\frac{K\mu\omega^2}{2u^2}$:

$$a = b = 2\pi\sqrt{\frac{\mu f}{\rho}}, \qquad V = \sqrt{\frac{\rho f}{\mu}} \qquad \text{and} \qquad \lambda = \sqrt{\frac{\rho}{\mu f}},$$

and in a distance of one wave-length either H or E is reduced to $e^{-2\pi} = 0{\cdot}001865$ of its initial value.

These expressions hold within an accuracy of 1 per cent., provided that $K\mu\omega^2/2u^2$ is not greater than 1/50th of $2\pi\mu\omega/\rho$ or $K\rho f < 0{\cdot}04u^2$ or $0{\cdot}36 \times 10^{20}$. Since ρ is about 25×10^9 for sea water, the maximum frequency for which the above expressions hold within an accuracy of 1 per cent. is $\frac{1{\cdot}44 \times 10^9}{K}$. The determination of K for such a highly conducting medium as sea water is extremely difficult, but the highest value which we have seen quoted is 80, from which the maximum frequency would be 18×10^6. As this covers all practical working frequencies, it would appear that the above formula may be used with confidence.

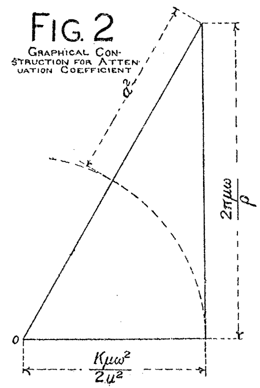

FIG. 2

GRAPHICAL CONSTRUCTION FOR ATTENUATION COEFFICIENT

In general the value of the attenuation coefficient can be readily obtained from the graphical construction, fig. 2, where $\frac{K\mu\omega^2}{2u^2}$ is set out horizontally and $\frac{2\pi\mu\omega}{\rho}$ vertically. The hypothenuse or resultant of these two quantities is therefore $\left\{\left(\frac{K\mu\omega^2}{2u^2}\right) + \left(\frac{2\pi\mu\omega}{\rho}\right)^2\right\}^{\frac{1}{2}}$ and the portion of it outside a circle with centre at O, and radius $\frac{K\mu\omega^2}{2u^2}$ is the square of the attenuation coefficient.

The following diagram, fig. 3, shows the attenuation, etc., plotted on logarithmic paper for sea water of resistivity 25ω per cm.[3] and exhibits the enormous attenuation which takes place even at short distances with moderately high frequency waves. For instance with a frequency of 10,000 ∼, corresponding to a wave-length of 30,000 metres in air, the intensity is diminished by absorption to $5{\cdot}7 \times 10^{-18}$ in only 100 metres, the velocity of propagation is only 158 kilometres per sec., and the wave-length 15·8 metres. At a frequency of only 1 ∼ per sec. the velocity of propagation in sea water is only about 1580 metres per sec., or approximately the velocity of sound in the same medium.

The above investigation not only shows the remarkable degree to which the velocity of propagation of electromagnetic waves is reduced by a conducting medium, and the unlikelihood of successful radio signalling between deeply submerged stations, but gives some clue as to the probable distribution of the magnetic field in the practical case of a stratum of water between a relatively poorly conducting bottom and a non-conducting atmosphere. For if we consider cylindrical electromagnetic waves emanating from the cable, it would appear that they should be propagated as cylindrical waves with low velocity in the water, but that on emergence above the surface and below the bottom the velocity would enormously increase, so that the lines of force should be of hour-glass shape with a narrow neck at the mid-depth of the water, thus making the magnetic field nearly horizontal above and below the water at long ranges, as the early experiments had already shown to be the case.

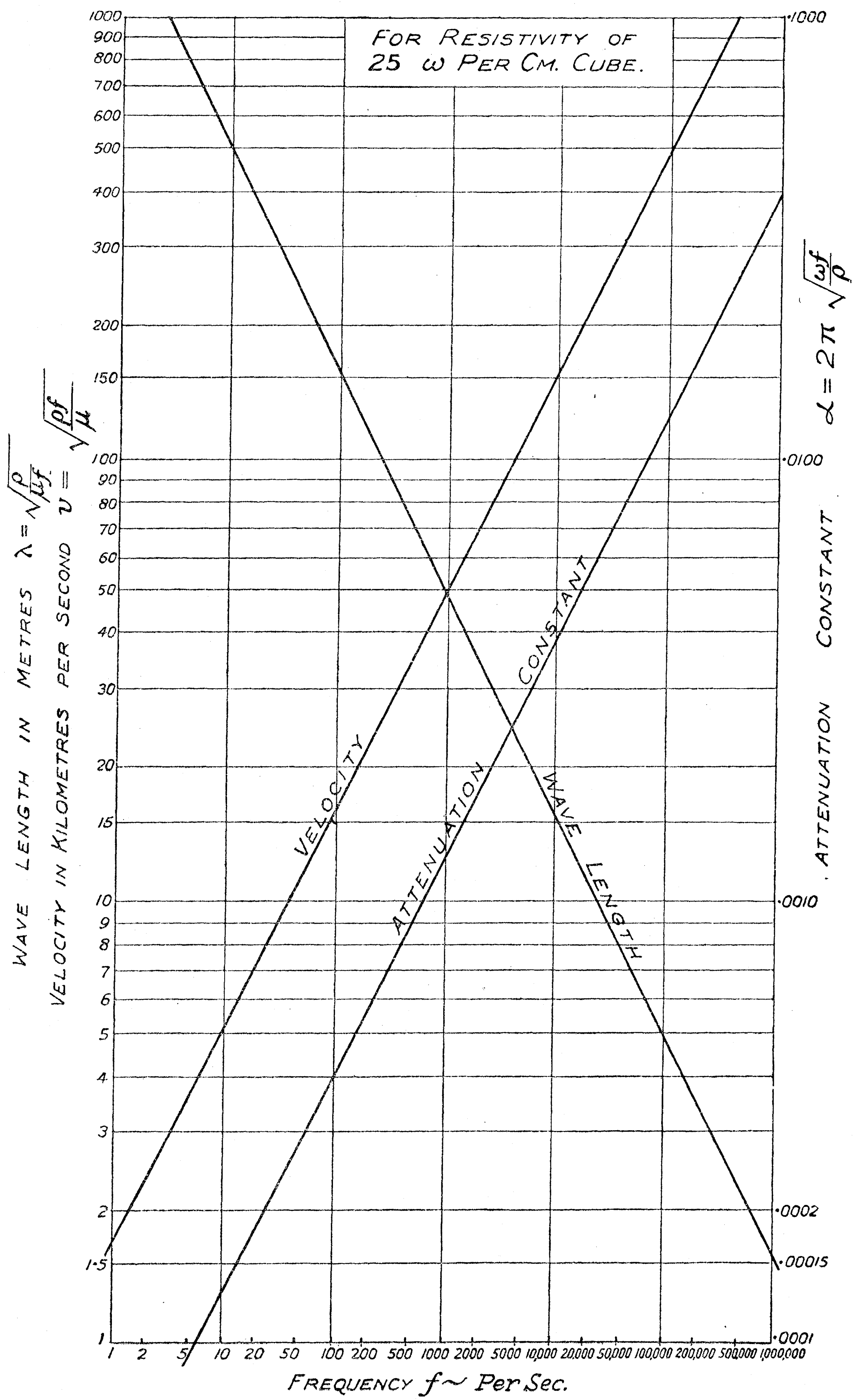

FIG. 3.—Charts showing variation of velocity of propagation, wave-length,

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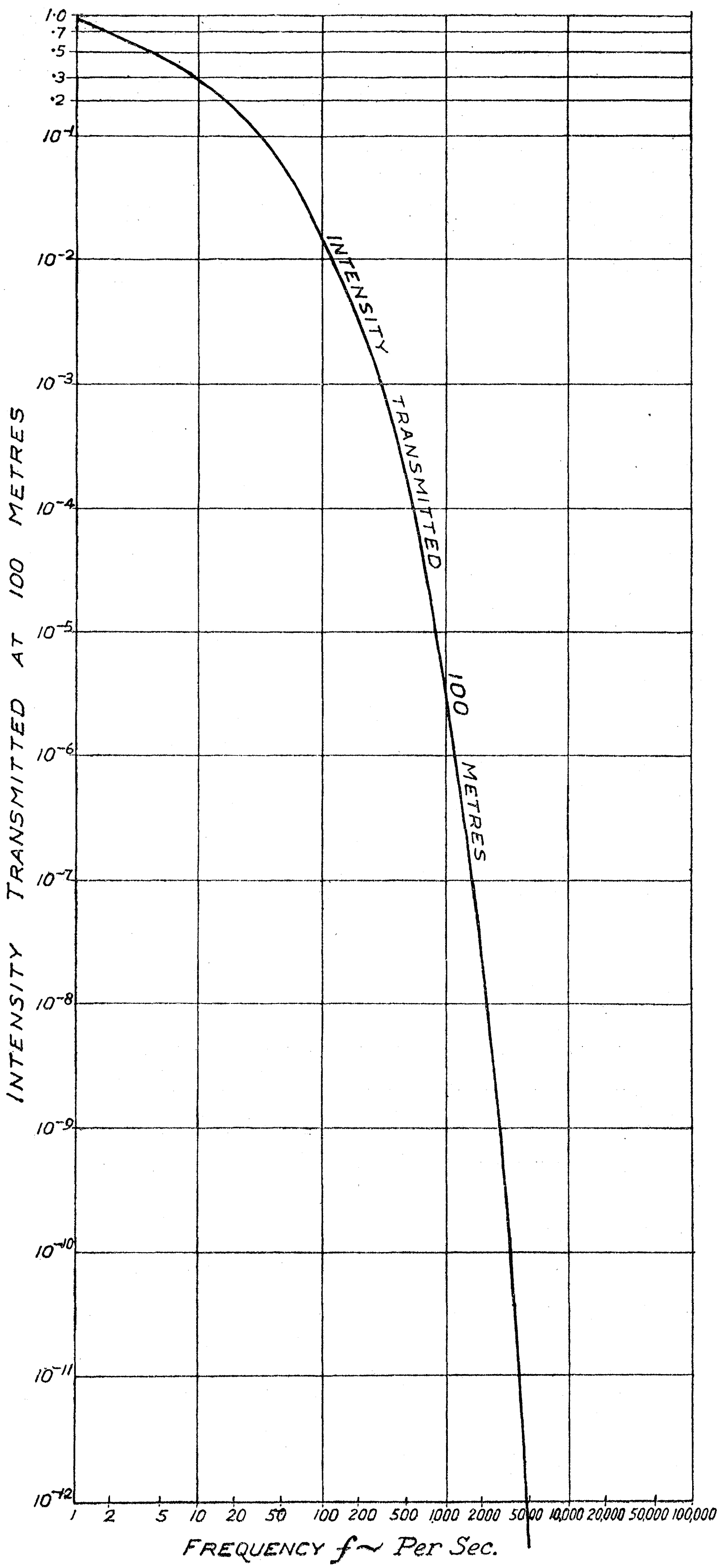

and attenuation, with frequency in sea water of resistivity 25 ω per cm³.

Cylindrical Conductor.—In the present investigation we are more concerned with the phenomena in the neighbourhood of the conductor than with plane waves. The fundamental equation for the current density will then be similar to that for E or H above, except that as curl $H = \frac{1}{r}\frac{\partial}{\partial r}(rH) = \frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r}$ in this case, we have to add the term $\frac{1}{r}\frac{\partial \Delta}{\partial r}$, where r is the radius from the centre of the cable core, giving us the well-known Fourier-Bessel type of equation $\frac{\partial^2 \Delta}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta}{\partial r} - \frac{4\pi\mu}{\rho}\frac{\partial \Delta}{\partial t} - \frac{K\mu}{u^2}\frac{\partial^2 \Delta}{\partial t^2} = 0$, which reduces to the plane wave equation when r is infinite.

This equation may be written in the form $\vartheta^2 \Delta = \left(\frac{4\pi\mu}{\rho}\dot{\Delta} + \frac{K\mu}{u^2}\ddot{\Delta}\right) r^2$, where $\vartheta = r\frac{\partial}{\partial r}$, and since it has been already shown that capacity effects are negligible at all working frequencies for a medium of such high conductivity as sea water, $\vartheta^2 \Delta = \frac{4\pi\mu}{\rho} r^2 \dot{\Delta}$. This is the well-known question for the "skin effect" in conductors solved by Lord KELVIN by the aid of his ber and bei functions, but a solution for a limited radius can be obtained by direct expansion in a series. Assuming sinusoidal supply of frequency $f = \omega/2\pi$, we may write $\Delta = f(r) \sin\{\omega t + \phi(r)\}$ where $f(r)$ and $\phi(r)$ are functions of the radius. Then, if we write θ for $\omega t + \phi(r)$, we have $\dot{\Delta} = \omega f(r) \cos\theta$, and

$\vartheta^2 \Delta = [\vartheta^2 f(r) - f(r)\{\vartheta\phi(r)\}^2]\sin\theta + [f(r)\vartheta^2\phi(r) + 2\vartheta f(r)\vartheta\phi(r)]\cos\theta = A\omega r^2 f(r) \cos\theta$ or

$$[\vartheta^2 f(r) - f(r)\{\vartheta\phi(r)\}^2]\sin\theta + [f(r)\{\vartheta^2\phi(r) - A\omega r^2\} + 2\vartheta f\phi(r)\vartheta\phi(r)]\cos\theta = 0,$$

where $A = \frac{4\pi\mu}{\rho}$.

For this relation to hold for all values of t and hence of θ.

$$\vartheta^2 f(r) - f(r)\{\vartheta\phi(r)\}^2 = 0, \text{ and } f(r)\{\dot{\vartheta}\phi(r) - A\omega r^2\} + 2\vartheta f(r)\vartheta\phi(r) = 0.$$

Case I. Current Flowing through Water between Electrodes.—If Δ_0 is the current density along the axis of a cylindrical body of water and expanding $f(r)$ and $\phi(r)$ in ascending powers of r we have $\vartheta r^n = nr^n$ and $\vartheta^2 r^n = n^2 r^n$; and by equating coefficients in the above equation we get

$$\Delta = \Delta_0\{1 + 0{\cdot}1625x^4 - 1{\cdot}0175 \times 10^{-4}x^8 + 1{\cdot}594 \times 10^{-6}x^{12} - 3{\cdot}663 \times 10^{-8}x^{16} + \text{etc.}\}$$
$$\times \sin\{\omega t + 0{\cdot}250x^4 - 1{\cdot}737 \times 10^{-3}x^6 + 3{\cdot}092 \times 10^{-5}x^{10} - 7{\cdot}35 \times 10^{-7}x^{14} + \text{etc.}\},$$

where $x = r\sqrt{A\omega}$ This series is only reasonably convergent for values of x up to 2, and over this range it agrees very closely with the empirical formula

$$\Delta = \Delta_0 \cosh 0{\cdot}17\, x^2 \sin(\omega t + 0{\cdot}25\, x^{1{\cdot}875}).$$

In sea water for which $\rho = 25 \times 10^9$ C.G.S., $A = 5 \times 10^{-10}$ approximately so that for $x = 2$ at a frequency of 500 ~ per sec. r is about 16 metres and at this distance from the cable $\Delta = 1 \cdot 23\ \Delta_0 \sin(\omega t + 52°)$, so that at this distance from the axis the current density would be 23 per cent. in excess of that at the axis, and its phase would be advanced 52°. By means of Bessels or ber and bei functions greater radii can be dealt with, but there is little value in doing so, as we are not dealing with large cylindrical bodies of water in practice.

Case II. Sea Current as return to Submerged Cable.—Taking the water as a cylindrical return of radius c to an axial cable and writing u for $\frac{r-c}{c}$ we have, if a_n is the coefficient of u^n in $f(r)$,

$$\vartheta f(x) = \Sigma_1^\infty \{na_n + (n+1)\, a_{n+1}\} u^n,$$

$$\text{and } \vartheta^2 f(r) = \Sigma_1^\infty \{n^2 a_n + (n+1)(2n+1) a_{n+1} + (n+1)(n+2)\, a_{n+2}\} u^n,$$

with similar expressions for $\vartheta\phi(r)$ and $\vartheta^2\phi(r)$, so that by introducing these expressions and the boundary conditions and comparing coefficients we get

$$\Delta = \Delta_c \left\{1 + g^4\left(\frac{u^4}{12} + \frac{u^5}{15} + \frac{23}{360}u^7 + \frac{17}{280}u^8\right) + \text{etc.}\right\}$$

$$\sin\left\{\omega t + g^2\left(\frac{u^2}{2} + \frac{u^3}{3} + \frac{u^4}{8} + \frac{u^5}{10} + \text{etc.}\right) + \text{etc.}\right\},$$

where $g = c\sqrt{A\omega}$, and u is reckoned negative $= \frac{c-r}{c}$. This series is only moderately convergent for values of c up to unity for which value the current density close to the cable is about 28 per cent. higher than at the distance $1/\sqrt{A\omega}$, while its phase is advanced about 51°. For $\rho = 25 \times 10^9$ and a frequency of 500 ~ per sec. this would occur for a radius c of 8 metres, but it would be of little value to extend the calculation to greater distances, as the cable lies on the bottom instead of being along the axis of a cylinder. The above calculations are, however, of value as giving some idea of the variations of current density and phase to be expected.

An important conclusion, however, that arises from the fundamental equations is that since the current density and phase depend upon $x = r\sqrt{A\omega}$ or upon $r\sqrt{\frac{\mu\omega}{\rho}} = \frac{r}{\lambda}$ it gives us a principle of similitude from which the phenomena for a given radius, frequency, or conductivity can be deduced from those observed with different values of these quantities. For example, if instead of employing sea water of resistivity 25 ohms per cm. cube we used mercury for which $\rho = 100$ microhms, the same phenomena would be found at 1/500th of the distance with the same conductivity, or at 1/5000th of the distance if the frequency were increased 100 fold. It would, therefore, be possible to conduct the whole of the tests in a mercury tank in the laboratory on a scale of about 1 foot to the mile, if it were only possible to use search coils of sufficiently small dimensions. For determining the field distribution in the upper air at moderately large distances, however, this arrangement is quite suitable, and a simple metallic sheet may then be substituted for the liquid. Mr. S. BUTTERWORTH has accordingly carried out a series of experiments upon these lines, and has also completed a mathematical theory of the field distribution in the air, taking the water as represented by a conducting lamina. This theory is given in a separate paper by Mr. BUTTERWORTH, together with a comparison with the experimental results obtained from the present investigation, and on a reduced scale with a metal sheet. Briefly, Mr. BUTTERWORTH's theory leads to three expressions for the field intensity depending on the ratio of the depth of water to the wave-length, as follows :—

(*a*) For depths less than 1/5 wave-length, if the radial distance from the cable r exceeds $\lambda^2/4d$ where λ is the wave-length $\sqrt{\frac{\rho}{f}}$ and d the depth, the field is in quadrature with the current and its intensity $H = \frac{\lambda^2}{20d}\frac{I}{r^2} = \frac{\rho}{20df}\frac{I}{r^2}$ so that it is inversely proportional to the square of the distance and to the depth and frequency, and directly proportional to the resistivity. The lines of force then take the form of circles passing through the cable and with their centres vertically above it.

(*b*) For depths between $\lambda/5$ and λ the above expression for H must be multiplied by the factor $z/\sqrt{\cosh z - \cos z}$ where $z = 4\pi d/\lambda$, and the quadrature relation ceases to hold.

(*c*) For depths exceeding a wave-length the above expression must be replaced by $H = \frac{2\sqrt{2}}{\pi}\frac{I\lambda}{r^2}\varepsilon^{-2\pi d/\lambda}$ where r is measured from a point on the surface vertically above the cable and should exceed three wave-lengths. In this case the lines

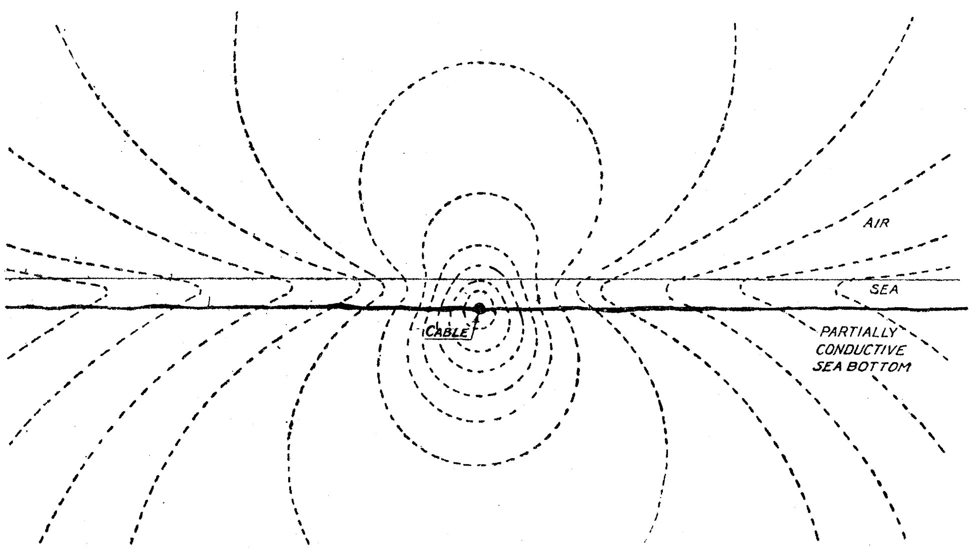

FIG. 4.—Probable distribution of magnetic field round submerged cable lying on partially conducting bottom.

of force beyond this distance from the cable approximate to the form of circles tangential to the upper and lower surface of the water vertically above and below the cable.

It will be seen, therefore, that for a given depth and resistivity the lines of force gradually change in form as the frequency rises, from circles encircling the cable at zero frequency to hour-glass form, and finally to two systems of circles tangential to the upper and lower surfaces at high frequencies. This is readily understandable from the great difference in the velocities of propagation in the air and water, which causes the waves to be greatly contracted along the horizontal diameter or plane of the lamina.

In the actual case of the sea with a bottom of high but not infinite resistivity, the velocity of propagation in the earth would be lower than in the air, and the lines of force would probably assume form similar to those indicated in fig. 4.

III. Preliminary Experiments.

Although this paper is mainly concerned with the recent systematic experimental investigation which has been carried out at Shandon, some account of the earlier experiments will be of interest, both for comparison with the more accurate measurements, and because they were taken under different conditions. They will be given briefly in chronological order.

In 1917, as mentioned above, Dr. F. B. Young observed in the course of some experiments in loop signalling at Parkeston Quay that the magnetic field at a considerable lateral distance from a submerged cable was nearly horizontal in direction, instead of being vertical as it should be on the ordinary theory of the circular field.

In January, 1918, the writer, with the assistance of Capt. H. A. Taylor and Mr. G. Stevenson, made some tests in Harwich Harbour, in order to ascertain the effect of absorption on the strength of the magnetic field in the centre of a submerged loop. An existing rectangular loop of nominal dimensions 300 yards by 200 yards laid between the gate ships at the mouth of the harbour was employed, and a carefully insulated search coil, having a mean area of 3·5 square metres and wound with 100 turns of No. 20 enamel-covered wire, was sunk as nearly as possible in the centre of this loop, which was of twin-core cable with the cores connected in series so as to make a two-turn loop. Current was supplied to this loop from a low-frequency alternator on board the wooden drifter "Hiedra," and the E.M.F. induced in the search coil was measured by connecting it through a rectifying commutator on the alternator shaft to a Sullivan moving coil galvanometer on one of the gate ships. In order to check the value of the deflections, a 1 ohm coil was connected in series with the galvanometer, and a cell, microammeter key, and high-resistance rheostat were connected across the terminals of this coil, so that by passing a current of a known number of microamperes through the coil, a P.D. of that number of microvolts was introduced into the galvanometer circuit without sensibly altering its resistance. Corrections made for the form factors of the alternating current and the interval of break at the rectifier almost exactly balanced, so that the galvanometer deflection was the same for a given P.D. in microvolts, either derived from the search coil or from the auxiliary C.C. circuit.

The coefficient of mutual inductance M, between a large rectangular loop of sides a and b and of n turns, and a small search coil of area A and n_2 turns at its centre, is $M = 8 \times 10^{-9} \mu A n_1 n_2 \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$ henrys, if μ is the permeability of the medium and there is no absorption. For the above loop and coil $M = 3 \cdot 34 \times 10^{-6}$ henry, assuming $\mu = 1$, so that for a current of 1 ampere in the loop at a frequency of 10 $\sim$ per second the induced E.M.F. $e_2 = 210$ microvolts R.M.S.

The following results were obtained :—

TABLE I.—Absorption at Low Frequencies (Depth about 30 ft.).

Frequency per sec.	Current in Loop R.M.S. amperes.	E.M.F. in Search Coil. R.M.S. Microvolts.		Ratio.
		Calculated.	Observed.	
10	1·13	240	238	0·99
20	2·20	923	828	0·90

These results cannot pretend to any great accuracy, as they were taken under very difficult conditions, and there was some doubt as to the exact dimensions of the loop, but they were sufficient to show that absorption was not of serious importance at these low frequencies. With a loop as inductor these results are not comparable with the Leader Gear results described later, as there was no diminution due to return current in the water. Incidentally it was found that by using an interrupted current of sonic frequency in the loop, and a horizontal search coil with telephones and amplifier on the ship, the position of the cable could be located and its course followed with considerable accuracy, and a survey of the loop was projected, but bad weather conditions intervened and the urgency of the practical problems prevented it from being carried out.

In the meantime the theory of propagation of plane electromagnetic waves in a conducting medium had been worked out, but no reliable data were available as to the electrical properties of sea water. As the methods previously employed for the determination of electrolytic resistance appeared open to question, the writer devised a method based on the Kelvin double-bridge principle which could conveniently be employed with his inductance and capacity testing bridge. The specimen of water to be tested was enclosed in a cylindrical glass tube of known dimensions, separate current and potential electrodes being employed at each end of the tube. Each current electrode consisted of a platinum disc mounted on a small platinum tube, and the potential electrodes were in the form of flat discs or flat spirals of platinum gauze mounted on platinum stems passing through and insulated from the current electrodes. This arrangement ensured a uniform current distribution over the whole length and section of the tube, and the potential electrodes were therefore lying in equipotential planes.

Measurements were made with this device by Mr. H. R. RIVERS MOORE in April, 1918, employing currents of frequencies up to 100,000 $\sim$ per sec. derived from alternators or a valve oscillator, a "ticker" being used in the telephone circuit for supersonic frequencies, and a three-valve amplifier when very small current densities were employed. The following results were obtained :—

(i) *Sample from Harbour Mouth, Harwich.*

Specific resistance, 27 ohms per cm.^{3} at 15° C.
Temperature coefficient, 3 per cent. per 1° C. (negative).

(ii) *Sample from Open Sea off Weymouth.*

Specific resistance, 24 ohms per cm.^{3} at 15° C.
Temperature coefficient, 3 per cent. per 1° C.

No change of resistivity was observable within the following limits :—

Current density, 0·1 to 15 milliamperes per square cm.
Frequency, 0 to 100,000 $\sim$ per sec.
Length of water column, 17 cm. to 90 cm.

The area of the water column was about 2·7 sq. cm.

The higher resistivity of the Harwich water was probably due to dilution by the rivers Stour and Orwell, and the table of absorptions, etc., given above was therefore calculated for a resistivity of 25 ohms per cm.^{3}

Subsequent measurements have confirmed these figures.

As regards capacity, the tests made with these four electrode tubes have shown fairly conclusively that up to frequencies of 1000 $\sim$ per second no appreciable capacity effects are to be anticipated. Further investigations have been made on this subject and will be published separately.

About June, 1918, Capt. J. C. MANSON commenced experimenting with Leader Gear at Parkeston Quay, and the writer urged the importance of an exact determination of the magnetic field round a submarine cable, proposing the employment of an A.C. potentiometer to measure the magnitude and place of the induced E.M.F. in the search coil. Preliminary tests were made with this arrangement by Mr. H. R. RIVERS MOORE, B.Sc., in August, 1918, proving its feasibility, and a cable was laid across Harwich Harbour for continuance of the research. Intervention of more pressing matters prevented the complete investigation from being carried out, but a rough survey of the magnetic field distribution just above the surface of the water was made by Messrs.

J. T. IRWIN and H. R. RIVERS MOORE, using an inclined coil and telephones, leading to a distribution such as shown in fig. 4 and confirming the expectations derived from theory.

In November, 1918, the writer proposed a form of Leader Gear employing visual indications, by means of alternating current relays connected to two inclined coils in the rigging of the ship, and actuating lamps to give the steering indications. Tests were immediately carried out on H.M.T. "William Inwood," on a cable laid down at Stokes Bay, and proved perfectly successful so far as sensitiveness was concerned, but the steering indications were ambiguous. It had not been expected that distortion of the field would be serious at the low frequency of 15 ~ per second employed, and the trouble was at first attributed to inaccurate pairing of the relays. After the removal of the Parkeston Quay station to Shandon in March, 1919, H.M.S. "Auricula" was fitted with inclined coils, each coil being of 4 turns of 7/18 cable, having a mean area of 525 square metres and inclination 30° to the vertical. The resistance of each coil was about 1 ohm and the relay used with it was wound to the same resistance.

Tests made with accurately paired relays showed that the lamps could easily be operated at half a mile distant from a cable carrying 7 amperes at 15 ~ per second in a depth of 16–20 fathoms, but the difficulty as regards steering indications persisted. It was, therefore, obvious that there must be serious distortion of the magnetic field, even at this low frequency, and the matter was investigated by measuring the E.M.F.'s developed in the two inclined coils at various ranges from the cable, by means of a carefully tuned vibration galvanometer, the frequency of the supply being maintained constant with the aid of a stroboscopic device. Fig. 5a shows the variation of the E.M.F.'s

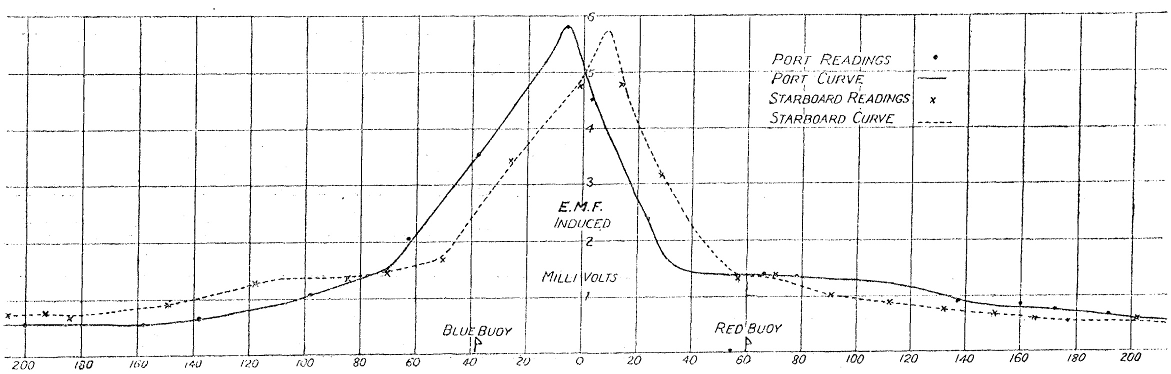

FIG. 5a.—E.M.F.'s induced in inclined coils on H.M.S. "Auricula."

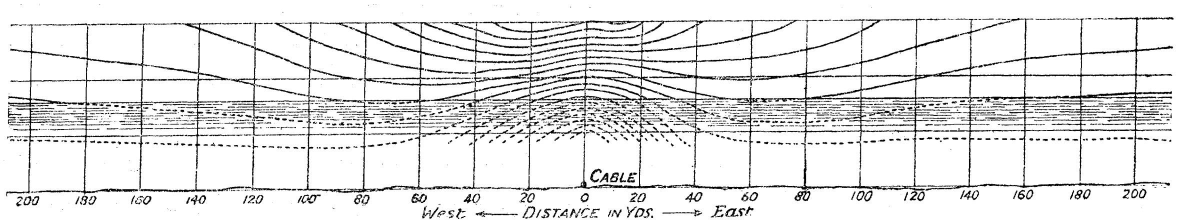

FIG. 5b.—Probable Field Distribution derived from above.

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observed, and fig. 5*b* the distribution of the magnetic field deduced from it, from which it appeared that the magnetic lines were approximately in circles immediately over the cable, but that at a lateral distance of about 70 yards they became horizontal and afterwards bent upwards at longer ranges. This result fully explained the difficulty of obtaining reliable steering indications, but was in conflict with the previous theoretical and experimental evidence in favour of a vertical field at moderate ranges. Similar experiments with a vibration galvanometer were therefore carried out with a horizontal and vertical coil on the bridge of H.M.S. "Auricula," with the results as shown in fig. 6.

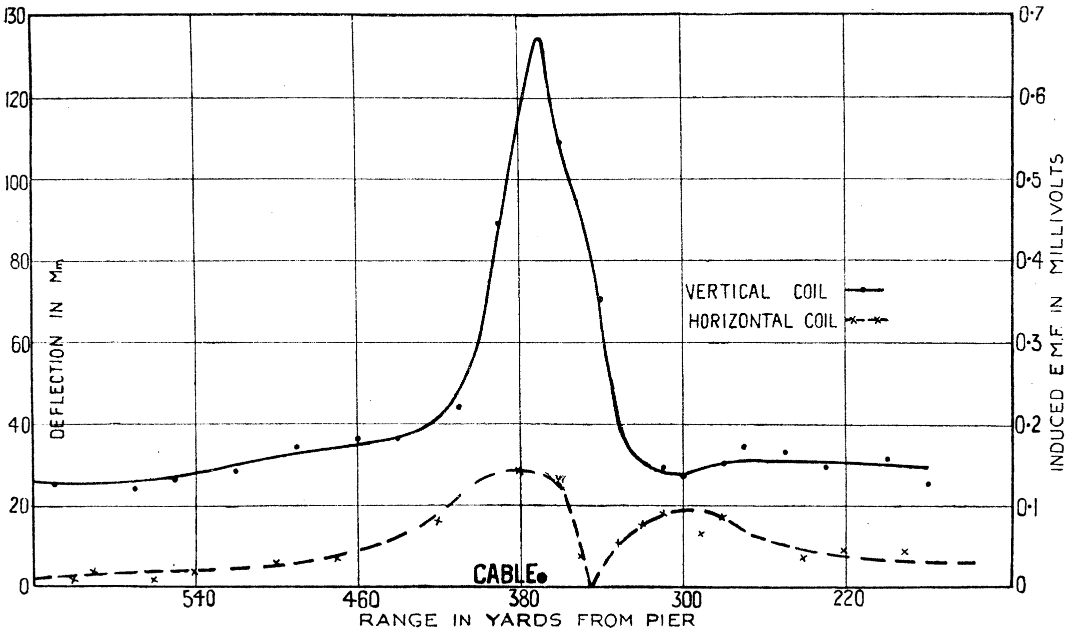

FIG. 6.—E.M.F.'s induced in horizontal and vertical coils on H.M.S. "Auricula."

It will be seen that the horizontal component of the field as measured by the E.M.F. in the vertical coil was always much greater than the vertical component, indicating that the field never departed greatly from the horizontal, and lending support to the conclusions from the inclined coils. The explanation of this phenomenon is apparently that the return current in the water concentrates in the hull of the ship, owing to its superior conductivity, thus producing a magnetic field encircling the hull and introducing a horizontal component into the field above it.

A few tests in which the E.M.F. of the two coils in series was measured indicated a considerable difference of phase between them at the regions of flexure of the lines of force, so that the field was evidently of a rotating character, but this was not followed up in the preliminary experiments.

Reference may be made at this point to the experiments made by Dr. H. LICHTE, of the laboratory of the Torpedo Inspection School at Kiel, in the summer and autumn of 1918. These experiments were carried out with submarine cables of different lengths and at depths of from 10 to 20 metres, with frequencies of 50, 500 and 1000 $\sim$ per sec.

In the majority of cases the measurements were made acoustically by comparing the intensity of the sound in a telephone connected through an amplifier to the search coils with that derived from a valve oscillator feeding a potentiometer device. This, of course, only enabled the intensity of the field to be determined, but in some cases phase measurements were made by the aid of a wireless train of the same note frequency derived from the alternator feeding the cable, and balancing the effect from the search coils against that of the wireless signal. Electrodes were also employed for measuring the electric fields.

The principal results obtained were as follows :—

(*a*) The distribution of the magnetic field above the surface was practically as predicted from general considerations of the skin effect in the water, the lines of force becoming vertical at a moderate lateral range from the cable and more and more nearly horizontal as the range was increased. According to the diagrams given in the paper, the ranges at which the field became vertical were approximately as follows :—

At 50 ~ = 36 metres, at 500 ~ = 17 metres, at 1000 ~ = 12 metres.

(*b*) Measurements on the effect of depth made on a cable laid in shelving water from 10 metres gave an exponential diminution of the field directly above the cable, the intensity being diminished to $\frac{1}{1\cdot 42}$ at 500 ~ and to $\frac{1}{1\cdot 51}$ at 1000 ~ for each 10 metres of depth.

(*c*) The intensity of the electric field parallel to the cable varied approximately as the square of the distance.

(*d*) The effect of inclination of the ship was small up to 80°.

Experiments were also made on the distorting effect of an iron hull, the conclusion being arrived at that this hull produced a field encircling it, and thus superposed a fairly strong horizontal component on the field above the deck. This may explain the effects found in the experiments on H.M.S. "Auricula."

IV. Systematic Experimental Research.

In view of the importance of obtaining accurate data on this subject, a proposal was put forward in 1918 for an investigation on a large and accurate scale ; and when the Admiralty experimental work was transferred to Shandon, early in 1919, the great suitability of the Gareloch for the purpose caused the scheme to be revived. Although a barge of the dimensions required for carrying a long mast and the necessary search coils could not be obtained, it was decided to make a start with an existing barge, the "St. Adrian," which was fitted up with a 40-feet mast carrying coils at top and bottom and a horizontal spar 10 metres long fitted with silver electrodes at its ends for measuring the current density in the water, attached to the lower end of the mast. The arrangement of the barge, mast, coils and electrodes is shown in fig. 7.

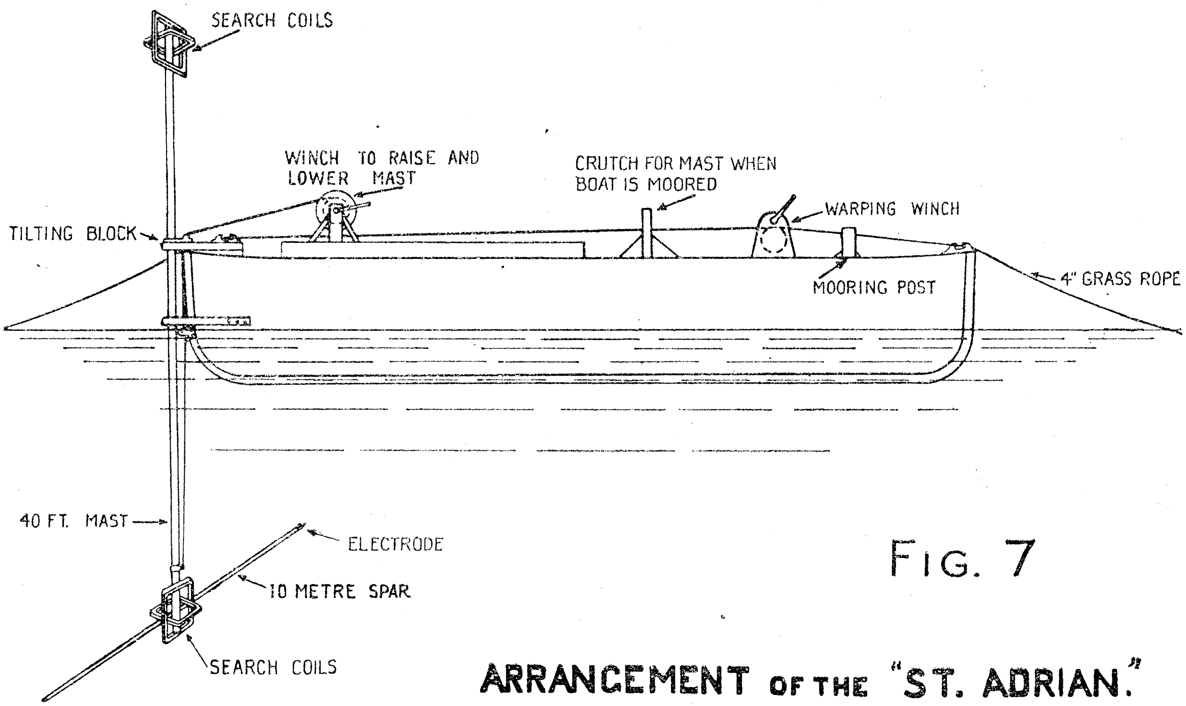

FIG. 7

ARRANCEMENT OF THE "ST. ADRIAN."

The cable employed in the experiments was a twin core armoured cable, each core consisting of 19/24 copper wire, and armoured with 22 No. 12 steel wires, the external diameter of the cable being $1\frac{1}{8}$ inch. It was laid out from the experimental station for a distance of about 3 miles down the Gareloch and in a depth of about $12\frac{1}{2}$ fathoms at low water at the working portion, the distance from the eastern shore of the loch being about 300 yards. From tests made upon this cable at a frequency of 500 ~ per sec. the following particulars were obtained :—

Effective	Resistance	per	mile,	8·25 ohms.
,,	Inductance	,,	,,	1·29 millihenrys.
,,	Capacity	,,	,,	1·67 mfd.
,,	Leakance	,,	,,	0·00522 mhos.

At a point in the cable immediately opposite Shandon pier, and about a mile from the station end, a junction box was inserted enclosing a 0·01 ohm standard non-inductive resistance, the terminals of which were connected by a similar twin cable to the laboratory on Shandon pier. This enabled the magnitude and phase of the current in the portion of the cable opposite Shandon pier to be measured on an A.C. potentiometer on the pier, and be employed as the standard of reference for all other measurements.

In order to traverse the barge at right angles to the cable, and to hold it in position during the time necessary for the measurement of the components of the magnetic field and the current density and phase in the water, a 600-yard coir rope was stretched

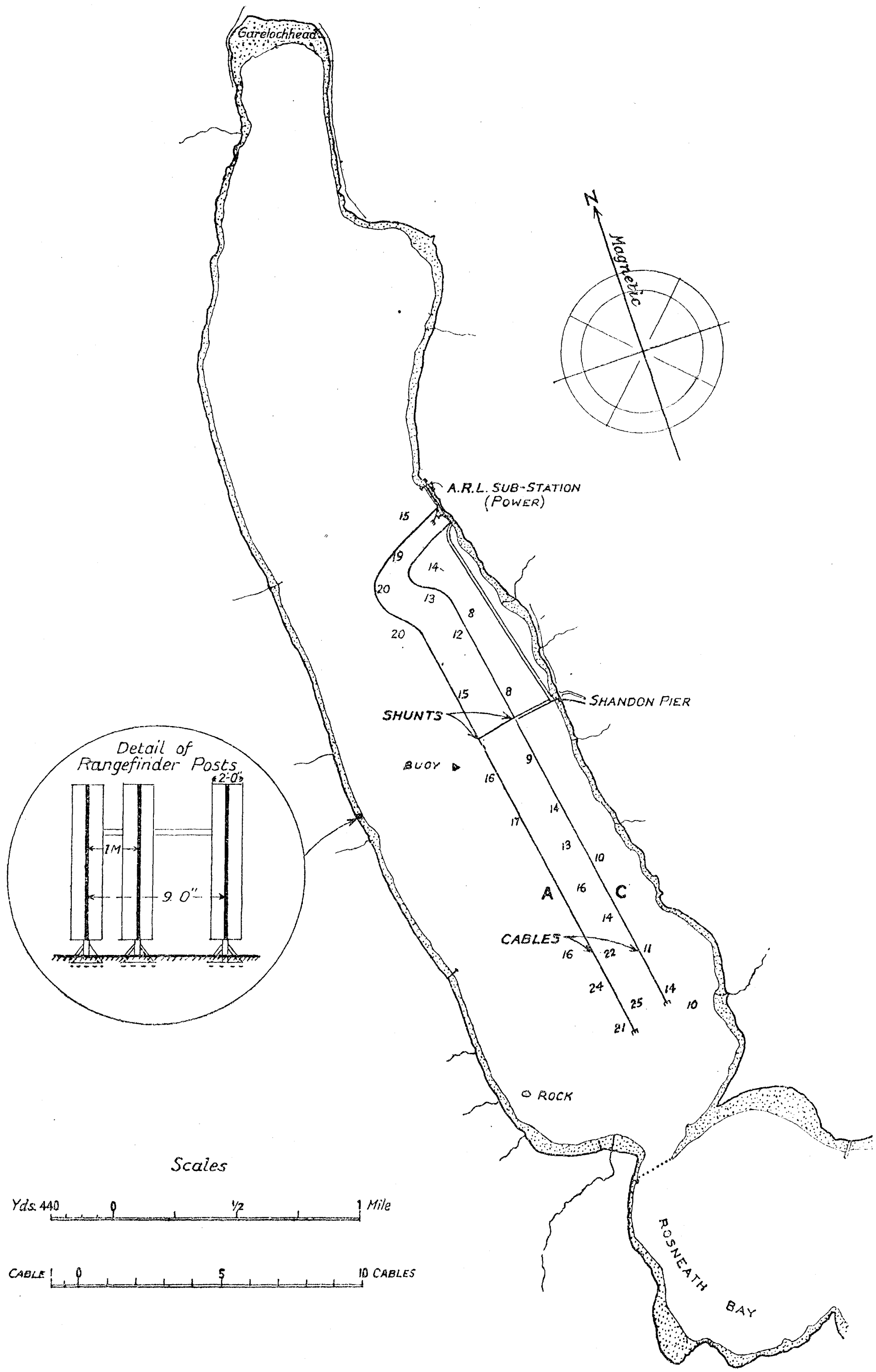

FIG. 8.—Plan of Gareloch, showing arrangement of cables, etc.

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from the pier directly across the loch to a buoy, and the barge was attached to this rope, so that by pulling on the rope from the barge the latter could be shifted at will.

In order to determine the range of the coils, etc., from the cable, a 1-metre and a 9-foot Barr and Stroud rangefinder were employed and installed in an observation hut on Shandon pier. The latter rangefinder was provided with a series of achromatic prisms enabling ranges to be measured to a minimum of 100 yards instead of the 1000 yards of the ordinary instrument. Various combinations of these prisms impart successive deviations of 0·003 radian, so that the true range is given by the formula

$$R = \frac{1000}{n + 1000/R'}$$

where R is the true range in yards

R′ the observed range on the rangefinder scale.

n the prism strength in multiples of 0·003 radian.

For checking and adjusting these rangefinders three vertical boards were set up on the opposite shore of the loch at intervals of 1 metre and 9 feet respectively on a line perpendicular to the cable, forming artificial infinity marks, and giving a standard direction for bearings. Fig. 8 is a plan of the Gareloch showing these arrangements.

A tide pole was set up at the end of Shandon pier, enabling the depth of water over the cable to be determined from its readings and the known soundings; the temperature and density measurements of the water were made during each set of readings. A new survey of the Gareloch was made by the Hydrographic Department, and the contour of the bottom along the line of traverse is shown in fig. 9 and also in the diagrams, figs. 17 to 22, showing the distribution of field, etc., obtained.

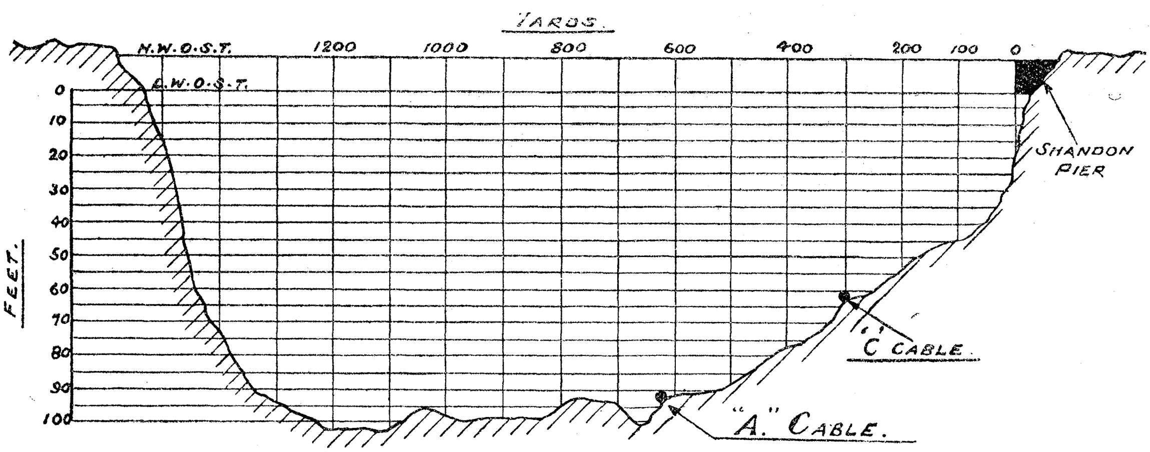

FIG. 9.—Section of Gareloch along experimental traverses.

For the measurement of the E.M.F.'s induced in the coils, and the P.D. between the electrodes, as well as the magnitude and phase of the cable current, a Drysdale-Tinsley alternating current potentiometer was employed, the connections to the coils and electrodes being made by an 8-core cab-tyred cable which was paid out from a drum on the barge as required. As the capacity of this cable was found to produce serious errors through resonance with the search coil at about 500 $\sim$, the tests from 120 $\sim$ upwards were made by transferring the potentiometer to the testing barge and exciting it from the shore through a twin core cable. A second 4-core cable was employed to effect connection to the shunts in the two junction boxes in the main cables.

Each search coil had a mean area of 9710 square cm. and was wound with 412 turns of No. 23 S.W.G., D.C.C. wire wound inside a rubber tube with vulcanised joint, so that the total effective area A of each coil was 400 square metres or 4×10^6 square cm. The E.M.F. induced for an R.M.S. field of B gausses is A B $\tilde{\omega} \times 10^{-8}$ volts, so that for a field intensity of 1 microgauss at 50 $\sim$ an E.M.F. of 12·56 microvolts is obtained which can just be measured on the potentiometer.

Instead of measuring the intensity and phase of the induced E.M.F., it was decided to determine the magnitude and phase of the horizontal and vertical components of the magnetic field directly by connecting the primary of a variable standard of mutual inductance in the main circuit of the potentiometer, and joining the secondary of the mutual inductance to the search coil and vibration galvanometer.

The primary of the mutual inductance was therefore traversed by a constant current of 0·05 ampere as indicated by the dynamometer of the potentiometer, the phase of which could be varied as required by turning the rotor of the phase-shifting transformer. The number of linkages in the search coil N = A B as before, while the number of linkages in the secondary of the mutual inductance $N' = M I \times 10^8$ where I is the primary current. When balance is secured on the vibration galvanometer $N' = N$ or $A B = 10^8 M I$, so that $B = 10^8 M I/A$, and is in phase with I. Since $I = 0{\cdot}05$ ampere and $A = 4 \times 10^6$ square cm.

$B = 1{\cdot}25$ M, and if M is in microhenrys, B is given in microgausses.

This arrangement has two important advantages.

(*a*) It gives the magnitude and phase of the magnetic field directly without any frequency calculation or quadrature relation.

(*b*) It isolates the search coil and galvanometer circuit from the potentiometer, and therefore eliminates the effect of capacity currents caused by the electrostatic capacity of the large search coils to earth if the potentiometer circuit is at an appreciable potential.

The latter point is of great importance, especially for high frequency measurements, as when large search coils are employed their electrostatic capacity may be quite sufficient to affect the vibration galvanometer seriously. The capacity of the 1-metre square coils used in this research would probably be of the order of 500 cm. or about 5×10^{-4} mfd., so that their capacity susceptance at a frequency of 50 $\sim$ would be $1{\cdot}57 \times 10^{-7}$ mho. As the potential of the main potentiometer may easily be several volts from earth, a capacity current of a few microamperes may flow to the coils and quite upset the balance.

By employing a mutual inductance of low mutual capacity, and also by earthing the mid-point of the primary, this effect can be practically eliminated, and a further important improvement is effected by using a differentially wound vibration galvanometer, with its two circuits interposed in the two leads to the search coil, as shown in fig. 10. This makes the arrangement perfectly symmetrical, so that any capacity

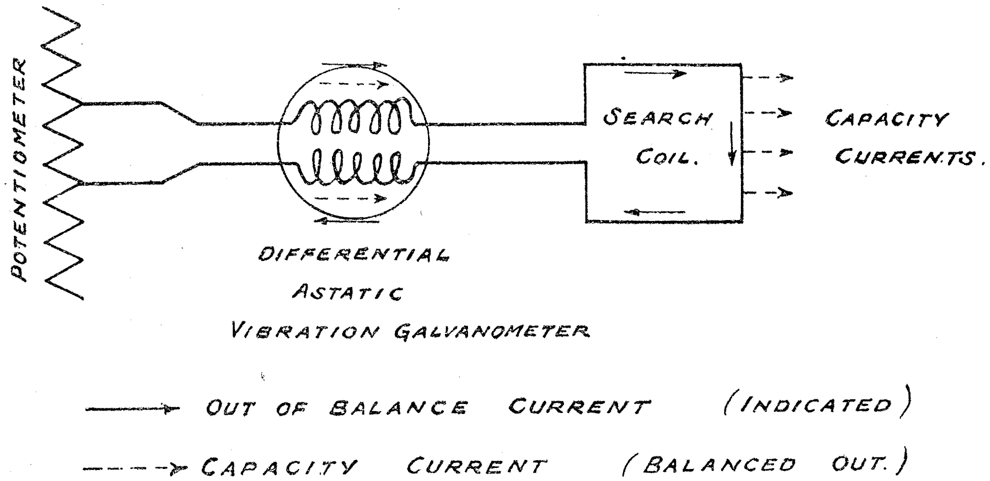

FIG. 10.—Differential Galvanometer Connections for eliminating capacity effects.

current divides equally between the two coils and is balanced, while any difference in the E.M.F.'s of the search coil and mutual inductance has full effect. The complete diagram of connections for measuring the cable current, the electrode P.D. and the components of the magnetic field above and below the water is shown in fig. 11.

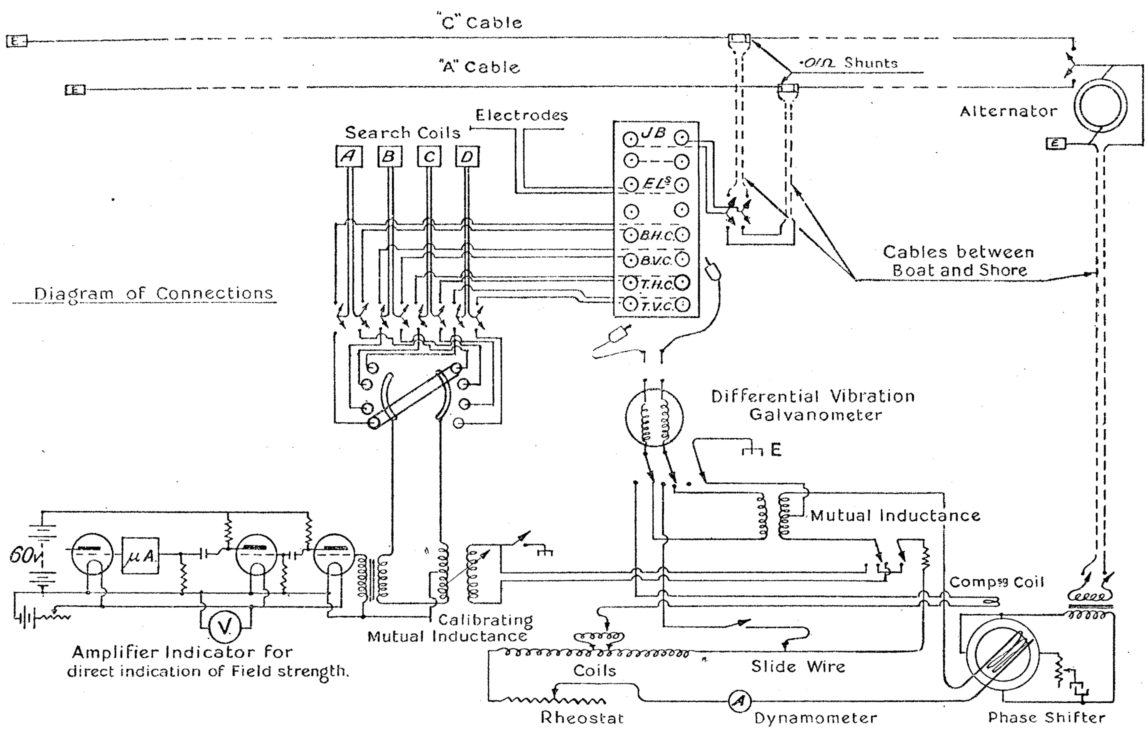

FIG. 11.—Complete diagram of connections for field and current distribution test.

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For the measurement of the density and phase of the return current in the sea, the P.D. between the electrodes was directly measured on the potentiometer. The leads from the two electrodes were carried along horizontally to the middle of the spar, and were then connected to two of the cores of the 8-core cable. This arrangement eliminates all inductive effects, so that the P.D. between the electrodes $V = l \rho \Delta$ where l is the length between electrodes, ρ the specific resistance and Δ the current density. In this case $l = 1000$ cm. and ρ has an average value of about 25 ohms per cm.^{3}, so that Δ microamps (per square cm.) $= 40$ V (volts) for this value of ρ. As the potentiometer will measure down to 10^{-5} volt, the current density can be measured to 4×10^{-10} amps. per square cm.

The current for exciting the potentiometer was supplied from the alternator feeding the cable, through a twin core armoured cable running down the eastern side of the loch close to the shore. No disturbance was to be apprehended from this cable, as the current taken by the potentiometer is only a small fraction of an ampere, and the twinning of the cores and shielding of the armouring eliminated the possibility of any appreciable external field.

Procedure during Tests.—The measurements were carried out by a series of traverses over the cable at a fixed frequency which was kept constant by hand regulation by the aid of a stroboscopic device. Three traverses extending for 200 yards or more on each side of the cable constituted a complete trial, the first being taken with the mast at its full height, with top coils about 30 feet above the surface and the bottom coils and electrodes about 10 feet below it; the second at half mast, with the coils respectively 20 feet above and 20 feet below the surface; and the third, with the top coils about 10 feet above and the bottom coils and electrodes about 30 feet below the surface. These three traverses enable the components of the magnetic field to be determined at six levels, three above and three below the surface, and the density and phase of the return current to be ascertained at three levels.

At each observation a flag was hoisted, upon which the range and bearing of the mast on the barge, the reading on the tide pole and the temperature of the water was taken by observers on the pier. Specimens of the water were also taken at intervals for conductivity tests.

V. Experimental Observations and Deductions.

As the mass of observations accumulated during the two years of the systematic research has been very considerable, only the most reliable and typical observations have been summarised in the plates 12 to 22, and one set of observations is given in Table II. Each of these plates shows the results of three traverses over the cable at a single frequency, exhibiting the variation of the magnitude and phase of the current density, and of the horizontal and vertical components of the magnetic field for three different heights and depths. In order to make all the diagrams comparable the observations have been corrected to an effective current of 10 amperes or 1 C.G.S.unit in this cable, and as a considerable portion of the current returns by the armouring at the higher frequencies, the ratio of effective current to core current was determined theoretically and experimentally for various frequencies as shown below and the corresponding factor used in correcting the observations. For the sake of comparison the curves showing the magnitude of the components of the magnetic field are accompanied by thinner curves showing the calculated values for these components if the water had been absent, viz.: $H_h = (I/d) \sin 2\theta$ and $H_v = (2I/a) \cos 2\theta$, where I is the effective current in C.G.S. units, d the vertical distance of the search coil above the cable, and θ the angle of inclination to the vertical of the line joining the coil to the cable. It will be seen that at the lower frequencies and especially below the surface, the maximum observed values of the components of the field are greater than the calculated values. This is readily understandable in the case of the underwater field, since if the return current were concentrated into a mirror image above the cable, the field close to the cable would be doubled. It was not expected that the field above the water would also be increased, but there is a certain amount of doubt about the value of the correction to be applied for the return current in the armouring at this frequency as no actual measuring was made of the ratio at 15 ∼ and the Oldenburg theory would appear to indicate that the ratio is nearly unity instead of 0·78 as taken from the experimental curve, fig. 26. With the higher ratio the maximum value of the above-water field agrees closely with theory. At the higher frequencies, however, the observed values of the field fall to a small fraction of the calculated value, showing the great effect of absorption.

An interesting feature of the horizontal field at the higher frequencies is that it shows decided secondary maxima, which are also indicated to a lesser degree, even at the lowest frequency of 15 ∼ per sec.

The phase relations of the two components of the field are of special interest and explain the distortion of the lines of force. Immediately above the cable the two components are approximately in phase with one another at all frequencies, and lag more and more behind the cable current as the lateral distance from the cable increases, but while the lag of the vertical component increases fairly gradually with the distance, that of the horizontal component increases almost suddenly to approximate anti-phase with the vertical component, and the curves appear to indicate that this difference of phase of about 180° would be maintained at greater ranges. This at once explains the distortion of the field, as the change of phase by 180° is equivalent to a reversal of direction of the horizontal component, thus causing the lines of force to slope away from the cable. At the greater ranges the horizontal component of the field is considerably greater than the vertical component so that the field becomes nearly horizontal.

As regards the character of the field, the equality of phase of the components close to the cable, and the opposition of phase at long ranges both indicate that the field

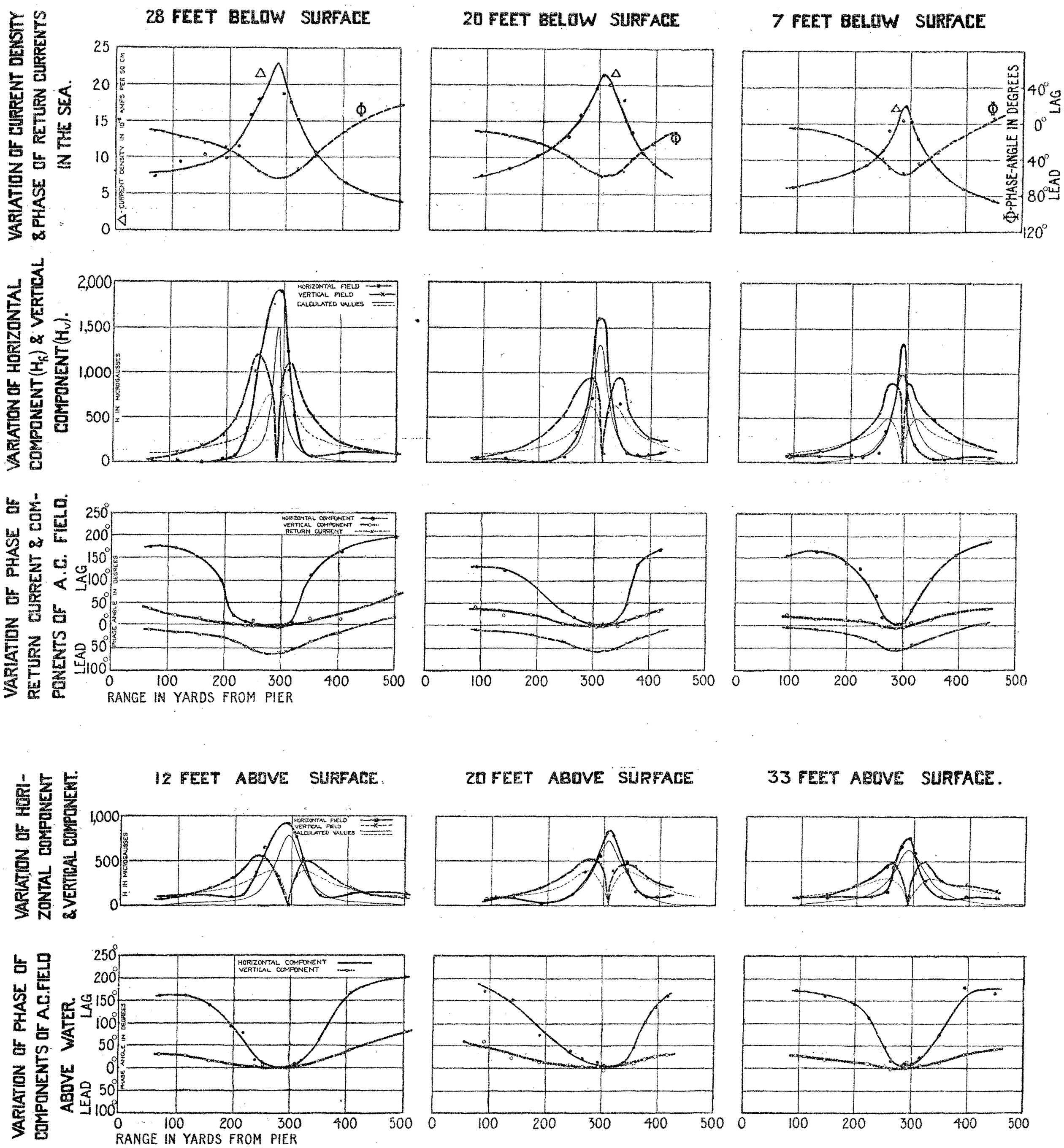

FIG. 12.—Magnitudes and Phases of Magnetic Field and Return Current at 15 ~.

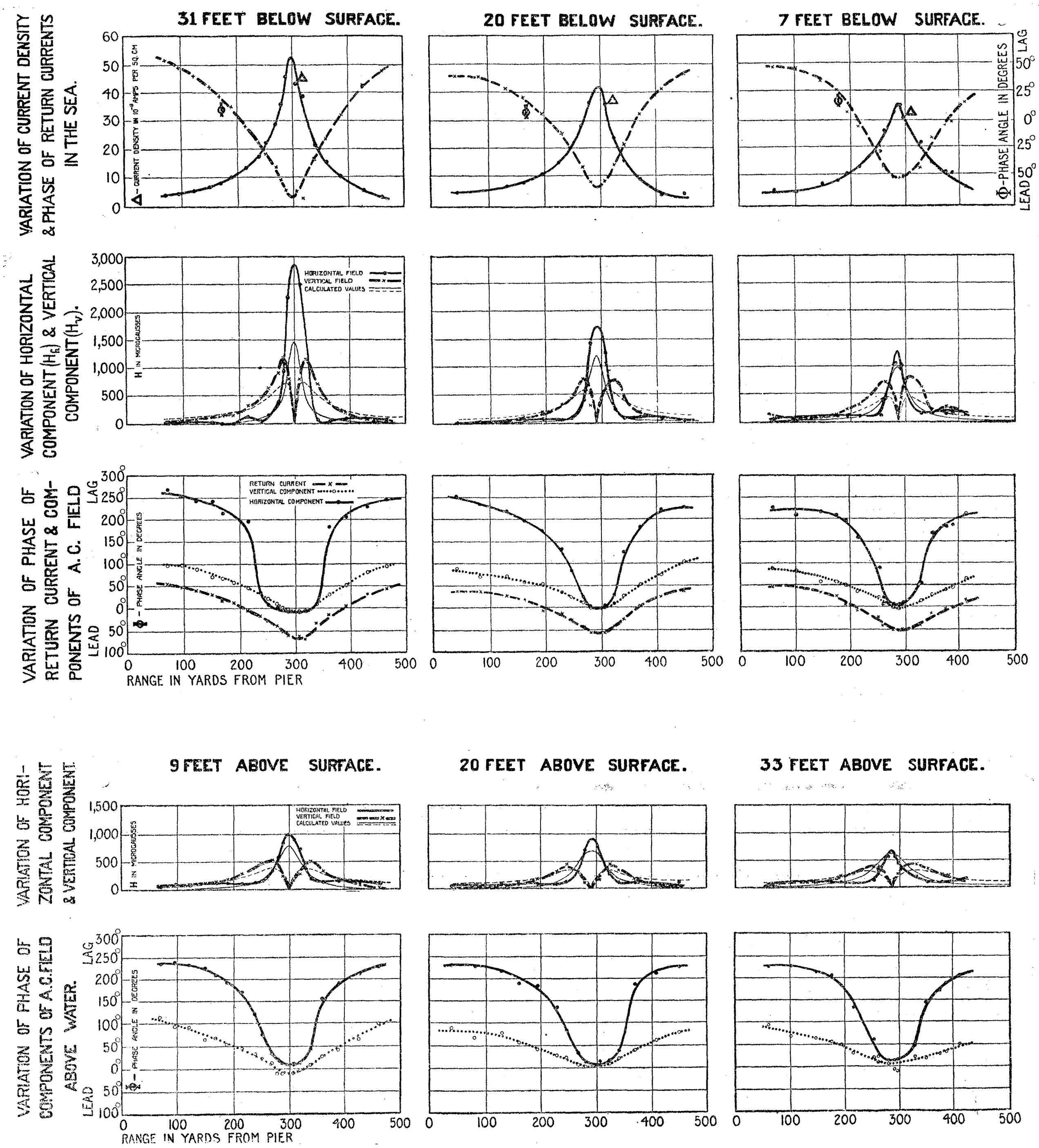

Fig. 13.—Magnitudes and Phases of Magnetic Field and Return Current at 50 ~.

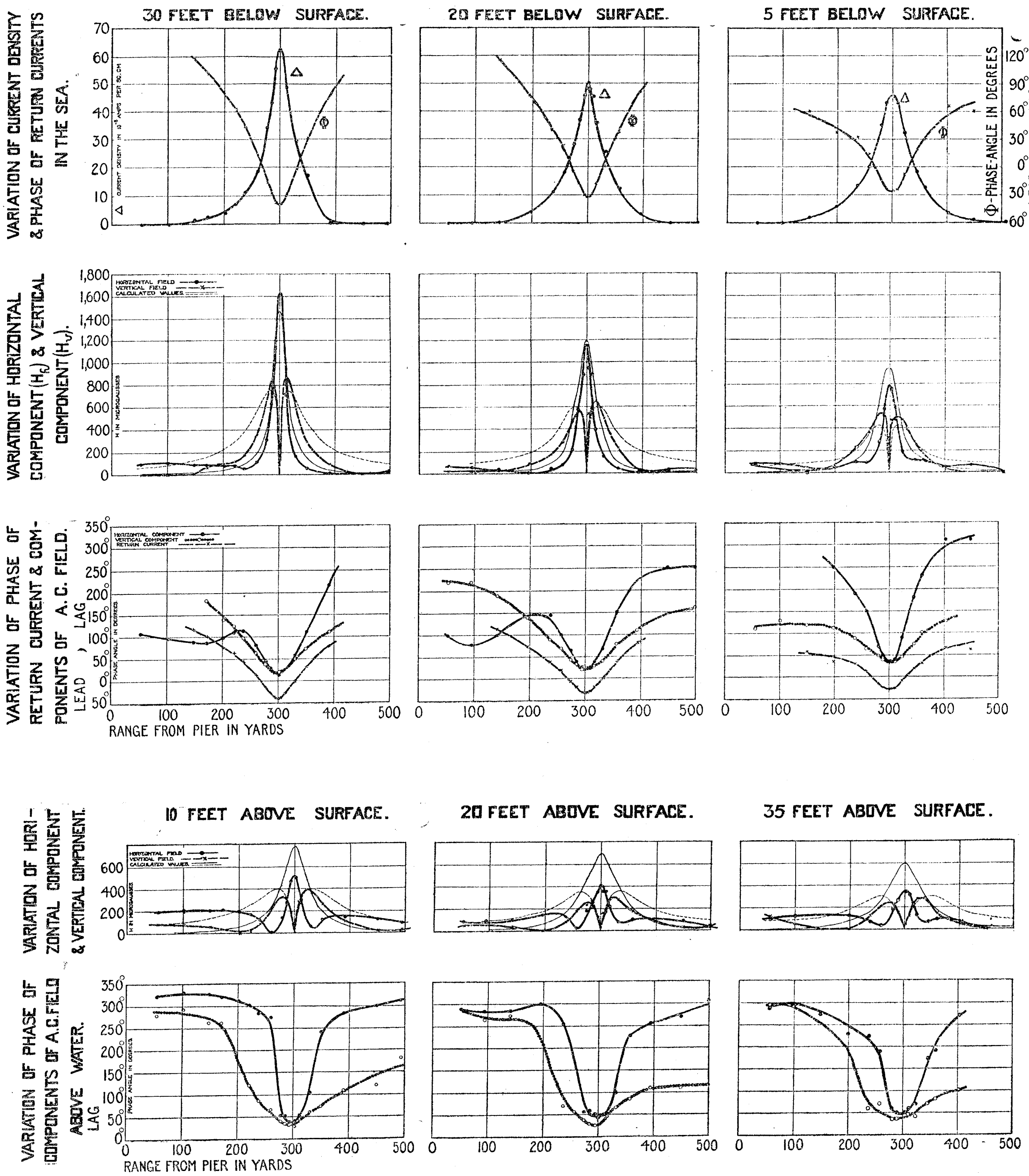

FIG. 14.—Magnitudes and Phases of Magnetic Field and Return Currents at 120 ∼.

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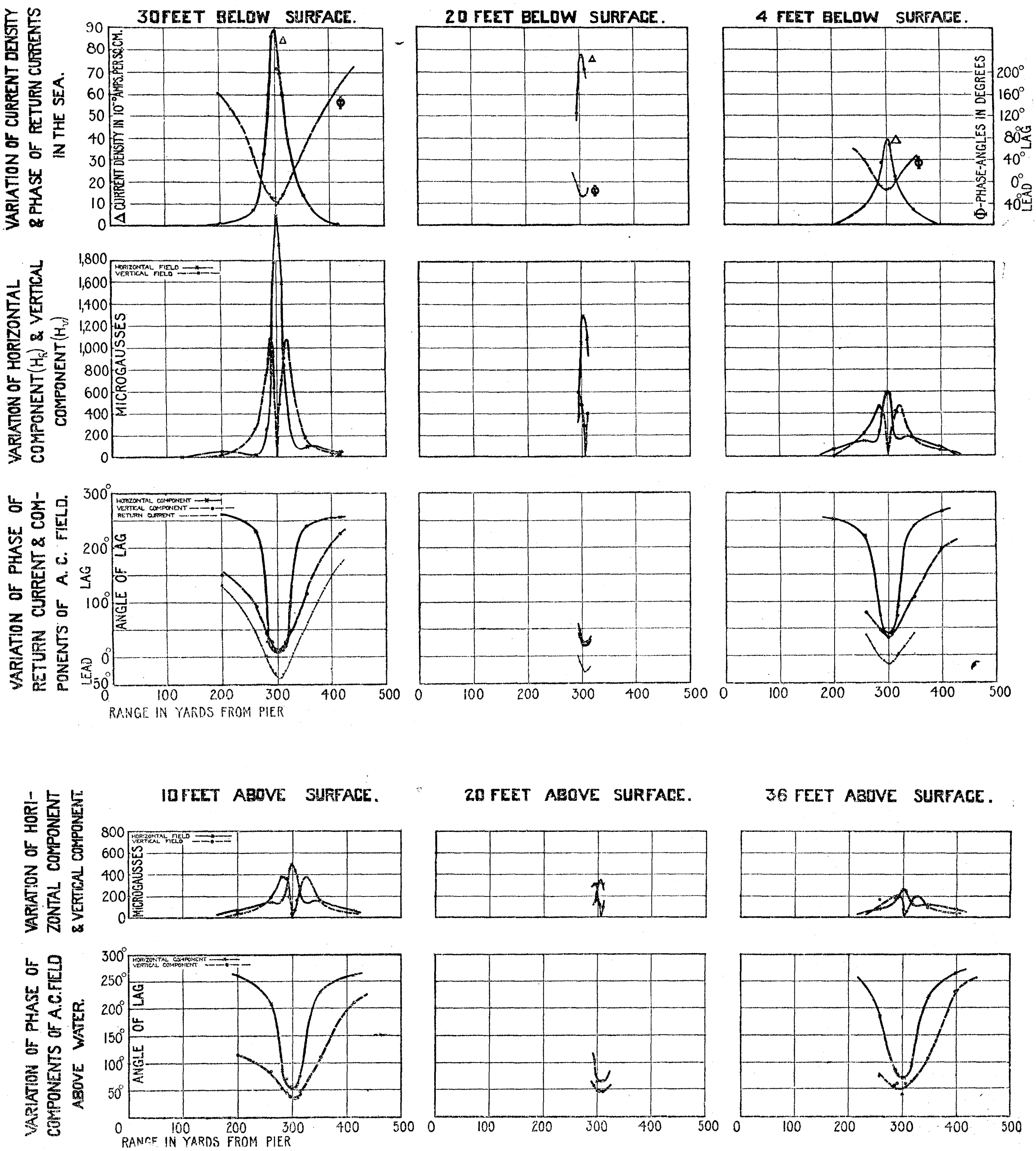

Fig. 15.—Magnitudes and Phases of Magnetic Field and Return Current at 250 ~.

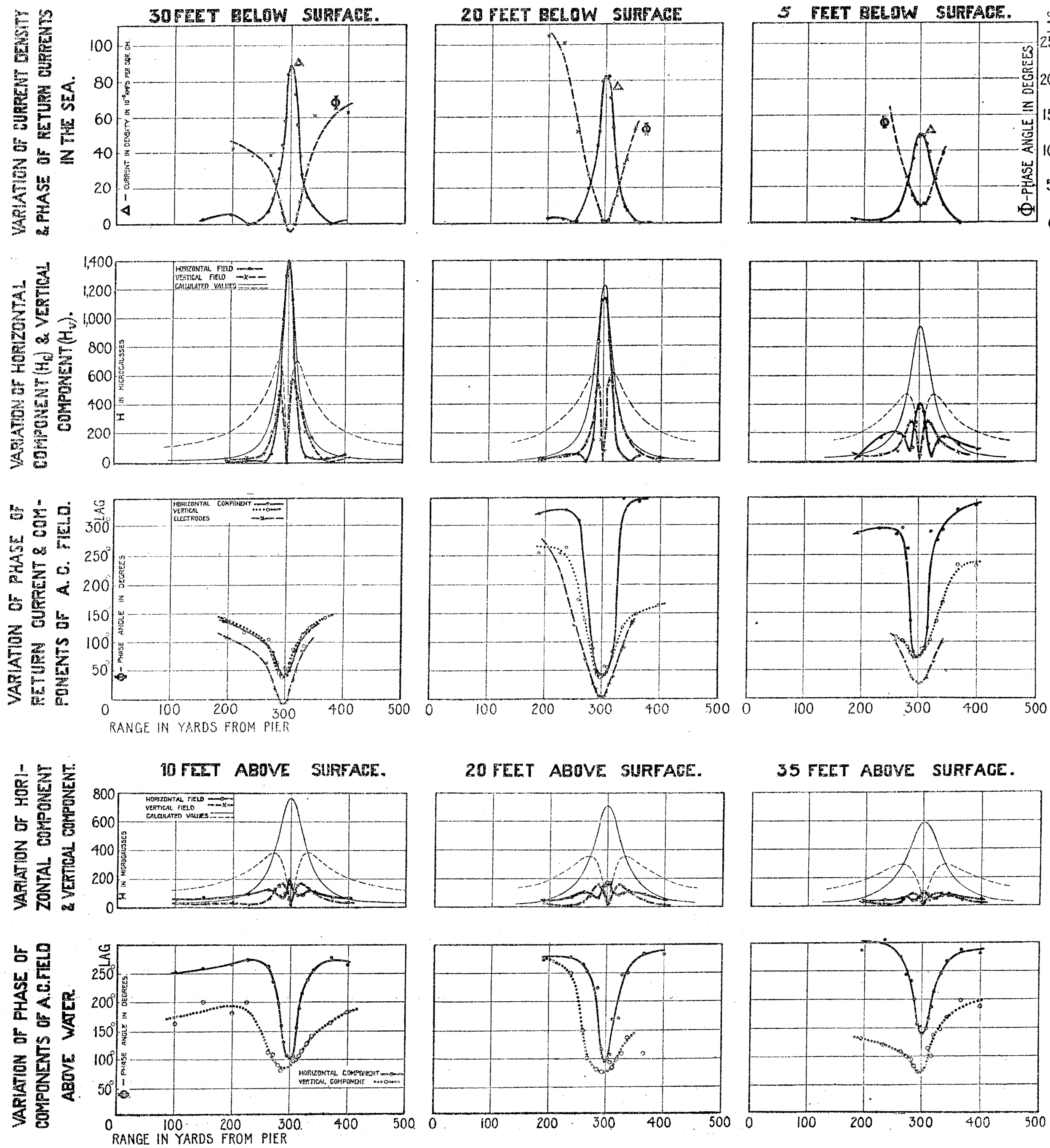

FIG. 16.—Magnitudes and Phases of Magnetic Field and Return Currents at 500 ∼.

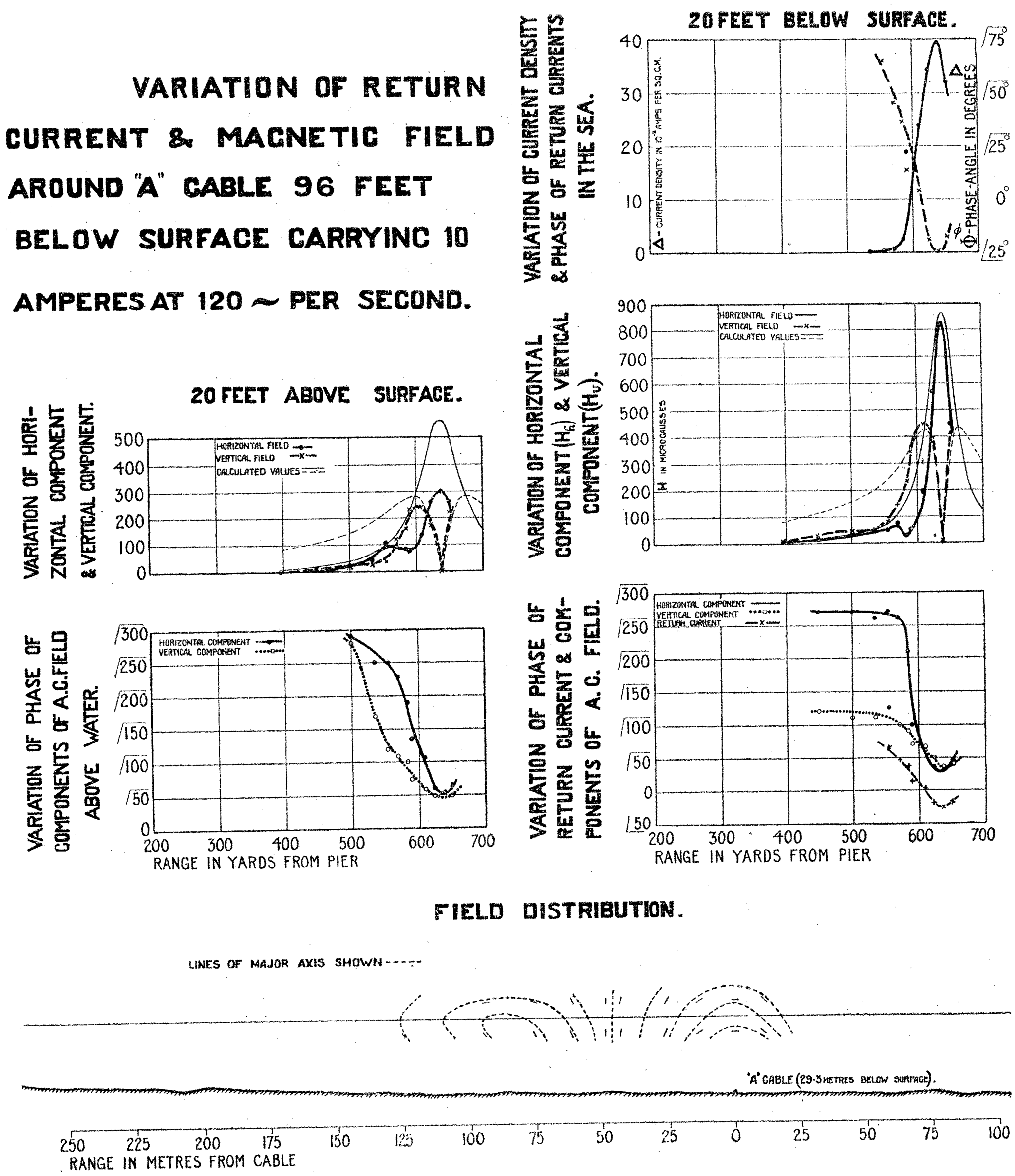
VARIATION OF RETURN CURRENT & MAGNETIC FIELD AROUND "A" CABLE 96 FEET BELOW SURFACE CARRYING 10 AMPERES AT 120 ~ PER SECOND.
20 FEET BELOW SURFACE.
VARIATION OF CURRENT DENSITY & PHASE OF RETURN CURRENTS IN THE SEA.
20 FEET ABOVE SURFACE.
VARIATION OF HORIZONTAL COMPONENT & VERTICAL COMPONENT.
VARIATION OF HORIZONTAL COMPONENT (Hc) & VERTICAL COMPONENT (Hv).
HORIZONTAL FIELD
VERTICAL FIELD
CALCULATED VALUES
VARIATION OF PHASE OF COMPONENTS OF A.C. FIELD ABOVE WATER.
VARIATION OF PHASE OF RETURN CURRENT & COMPONENTS OF A. C. FIELD.
HORIZONTAL COMPONENT
VERTICAL COMPONENT
RETURN CURRENT
RANGE IN YARDS FROM PIER
FIELD DISTRIBUTION.
LINES OF MAJOR AXIS SHOWN
"A" CABLE (29·3 METRES BELOW SURFACE).
RANGE IN METRES FROM CABLE

FIG. 17.

Table II.—Traverse No. 3. Field and Current Distribution round Submarine Cable. Surface. Bottom Coils and

Range in Yards from Pier.	Bearing.	Corrected Range Perpendicular to Cable.		Depth of Water Above Cable.		Current Density Δ Amps. per Sq. Cm. × 10^{-8}.	Bottom Coils		
		Yards.	Metres.	Feet.	Metres.		H_h Microgauss.	H_v Microgauss.	θ
473	12°	462	423	69·6	21·24	3·33 ⌈45°	66·75 ⌈246°	29·72 ⌈95°	151°
439	13°	427	391	70·7	21·6	5·56 ⌈32°	90·4 ⌈228°	92·15 ⌈81°	147°
398	12°	389	356	71·3	21·8	10·05 ⌈6·4°	97·2 ⌈206°	207·8 ⌈53°	153°
367	11°	360	330	71·6	21·8	15·85 ⌊14·8°	62·5 ⌈184°	384·5 ⌈32°	152°
343	10°	338	310	—	—	21·69 ⌊32·9°	69·3 ⌈10°	728 ⌈12°	—2°
319	2°	318	291	73·0	22·25	38·89 ⌊68·1°	1514 ⌊4·5°	1174 ⌊6°	1½°
309	7°	307	281	73·8	22·5	43·75 ⌊61·8°	2484 ⌊6°	806 ⌊9°	3°
		Cable about 300 yards from pier							
299	16°	287	262	74·6	22·8	46·02 ⌊60·8°	2265 ⌊5°	806 ⌊3°	—2°
291	17°	278	254	75·0	22·9	36·0 ⌊51°	945 ⌊1°	1205 ⌈3°	—4°
278	16°	268	245	75·4	23·0	28·72 ⌊40·1°	391 ⌈6°	925 ⌈11°	—5°
264	18°	251	230	75·7	23·1	22·49 ⌊28·1°	155·4 ⌈9°	639 ⌈22°	—13°
248	16°	238	218	75·9	23·2	17·67 ⌊16·4°	40·6 ⌈33°	453 ⌈35°	—2°
229	20°	215	197	76·1	23·2	13·62 ⌊3·8°	129·3 ⌈197°	328 ⌈47°	+150°
202	21°	189	173	76·2	23·24	10·27 ⌈12·8°	O	208·6 ⌈58½°	—
180	20°	169	155	76·2	23·24	8·43 ⌈22°	17·24 ⌈213°	163·7 ⌈69°	144°
161	21°	150	137	76·1	23·2	7·02 ⌈31°	30·25 ⌈241°	138·5 ⌈71°	170°
130	20°	122	111·5	76·0	23·2	5·57 ⌈41°	45·25 ⌈242°	88·6 ⌈87½°	154½°
99	—	96	88	75·75	23·1	4·57 ⌈48°	23·31 ⌈251°	77·75 ⌈98½°	152½°
71	6°	70·5	64·5	75·5	23·0	3·92 ⌈54·7°	34·6 ⌈264°	49·5 ⌈97°	167°

is practically of an alternating character; but over the short range during which the phase of the horizontal component changes rapidly it is of an elliptical rotating character, and in a few cases, notably at 35 feet, above the surface with a frequency of 500 ∼ and

Traverse over Cable carrying 10 eq. Amperes at 50 ∼ per second. Top Coils 9 ft. above Electrodes 31 ft. below Surface.

(Below Surface).				Top Coils (Above Surface).						
α	H max. Micro-gauss.	H min. Micro-gauss.	ϕ	H'_h Microgauss.	H'_v Microgauss.	θ'	α'	H′ max. Micro-gauss.	H′ min. Micro-gauss.	ϕ
158	71	13·66	174·5°	122·7 ⌈230°	47·3 ⌈101°	129°	165	126·6	35·55	175·7°
134	124	36·85	163°	141·9 ⌈213°	92·7 ⌈67°	146°	149	162·4	43·6	170·6°
114	226·4	40·65	157·4°	158 ⌈192°	209·6 ⌈44°	148°	122	253·2	70·25	159·0°
98	392·5	309·1	152·5°	145·6 ⌈157½°	343 ⌈29°	128½°	77	353·1	118·3	168°
84	734	11·94	—2·2°	194·6 ⌈45°	513·5 ⌈9°	36°	72	537	109·5	32·1°
38	1911	53·4	0·8°	727 ⌈14½°	285·3 ⌊2°	16½°	21	777	74·5	1·9°
18	2609	37·87	0·3°	895 ⌈12½°	168·4 ⌊9°	21½°	10	904	59·2	0·7°
20	2400	65·6	—2·2°	847 ⌈13°	248·7 ⌈9°	4°	16	888	109·7	0·4°
52	1266	53·4	—3·7°	592 ⌈23°	517 ⌈9°	14°	41	781	93	6·1°
67	1004	23·9	—4·25°	335 ⌈35°	543 ⌈14½°	20½°	59	544	102	14·8°
77	656	31·1	—12°	155·3 ⌈77°	485 ⌈24½°	52½°	22	493	120·6	29·6°
85	454	6·56	—2·0°	133·6 ⌈122°	407 ⌈34°	88°	93	406	134·3	87·7°
92	328	9·26	150°	109·9 ⌈171½°	284·7 ⌈46°	125½°	104	292	87·9	129·2°
—	208·6	O	—	131·6 ⌈193½°	198 ⌈56°	137½°	120	225	78·6	149·1°
95	164·4	9·27	154°	133·6 ⌈210°	155·1 ⌈69°	141°	130	193·3	67·6	157·2°
102	141·4	5·34	170·2°	114·5 ⌈227°	125·5 ⌈67°	160°	132	166·9	30·25	169°
116	97·9	5·61	159·2°	97·2 ⌈233½°	99·4 ⌈93°	140½°	134	133·4	39·1	158·7°
	80·3	10·4	154·4°	103·7 ⌈238½°	75·8 ⌈94½°	144°	146	121·5	37·8	168°
125	60	6·56	171·2°	71·75 ⌈235°	55·25 ⌈101½°	133½°	137	79·3	34·3	164·3°

range of about 50 yards, the major and minor axes of the ellipse are nearly equal, indicating that the field is almost purely rotatory. In the diagrams showing the field distribution the direction of the major axis only has been indicated.

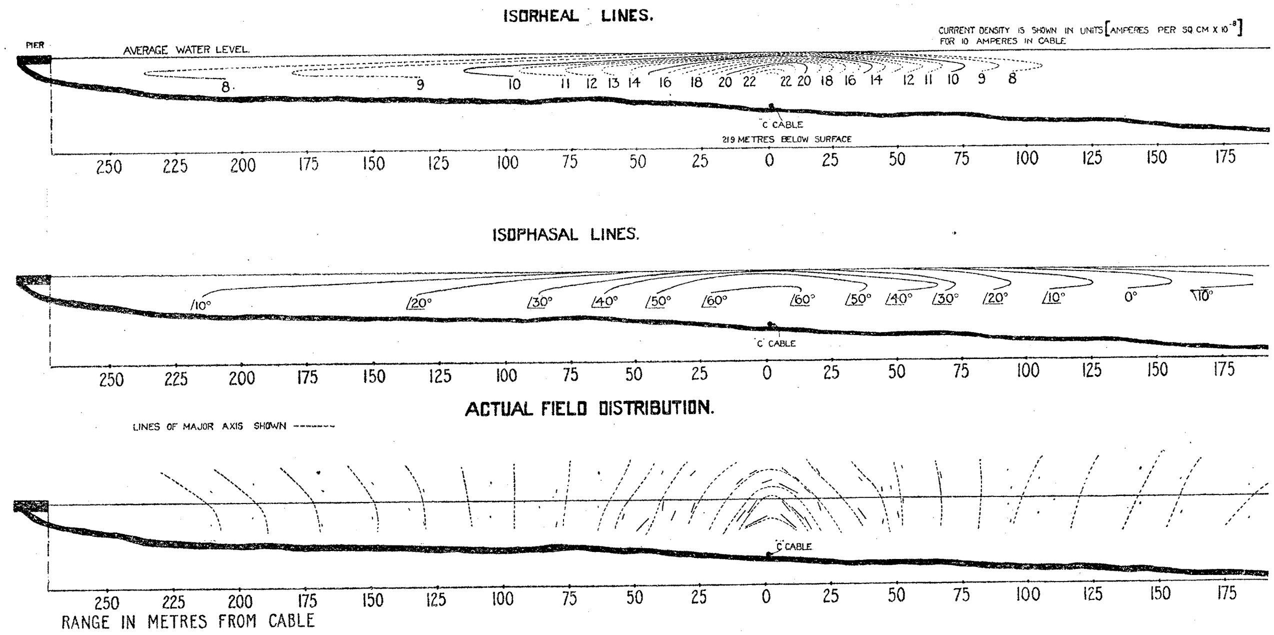

FIG. 18.—Current and Field Distribution around Cable carrying 10 amperes at 15 ~ per second.

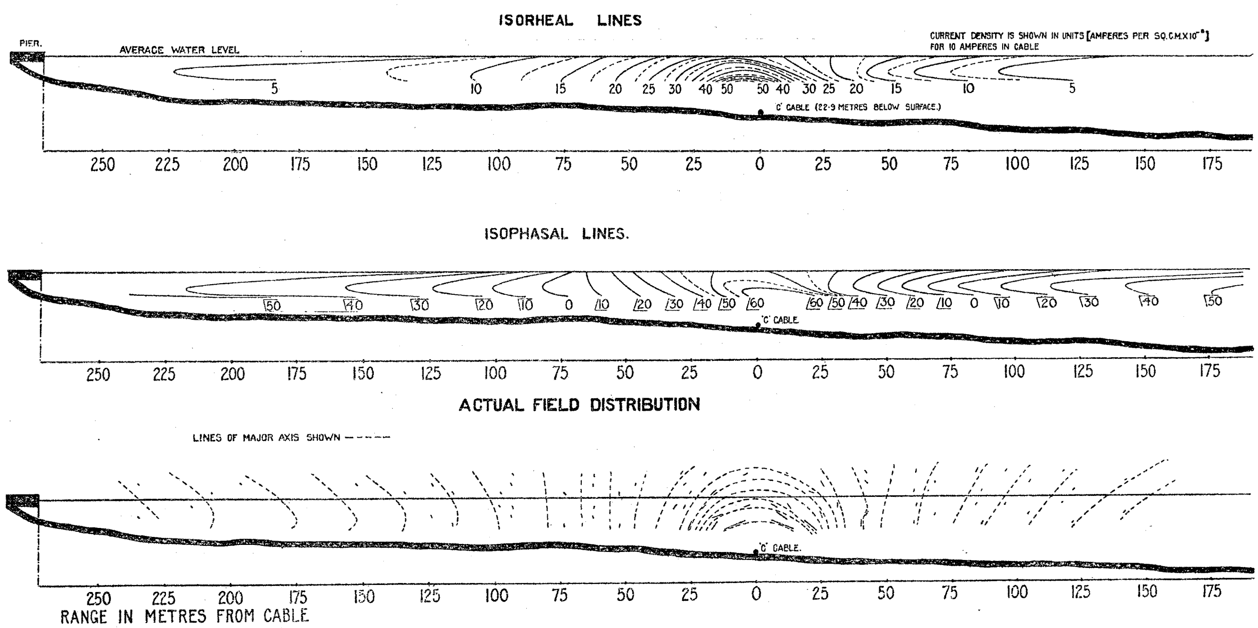

FIG. 19.—Current and Field Distribution around Cable carrying 10 amperes at 50 ~ per second.

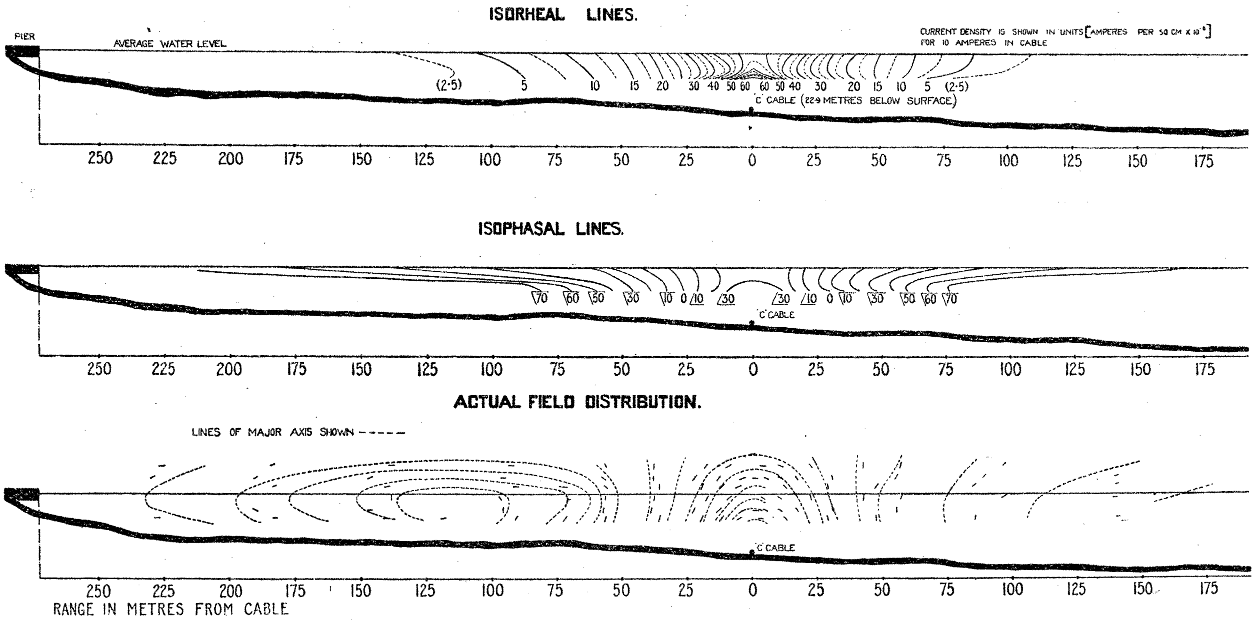

FIG. 20.—Current and Field Distribution around Cable carrying 10 amperes at 120 ∼ per second.

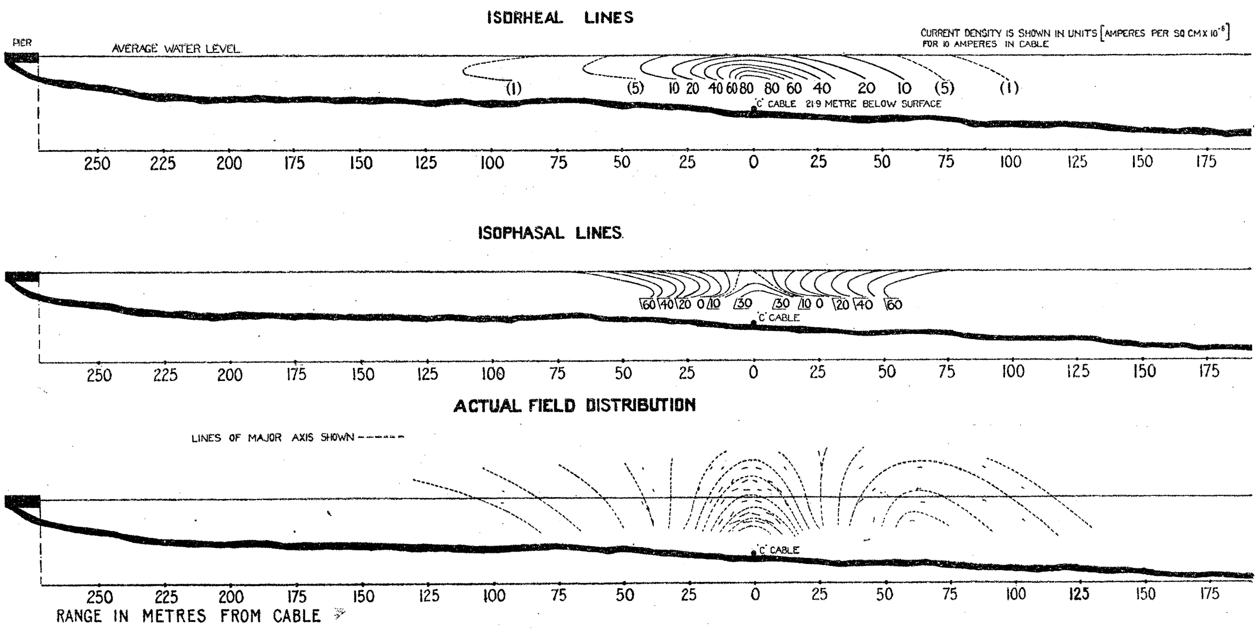

FIG. 21.—Current and Field Distribution around Cable carrying 10 amperes at 250 ∼ per second.

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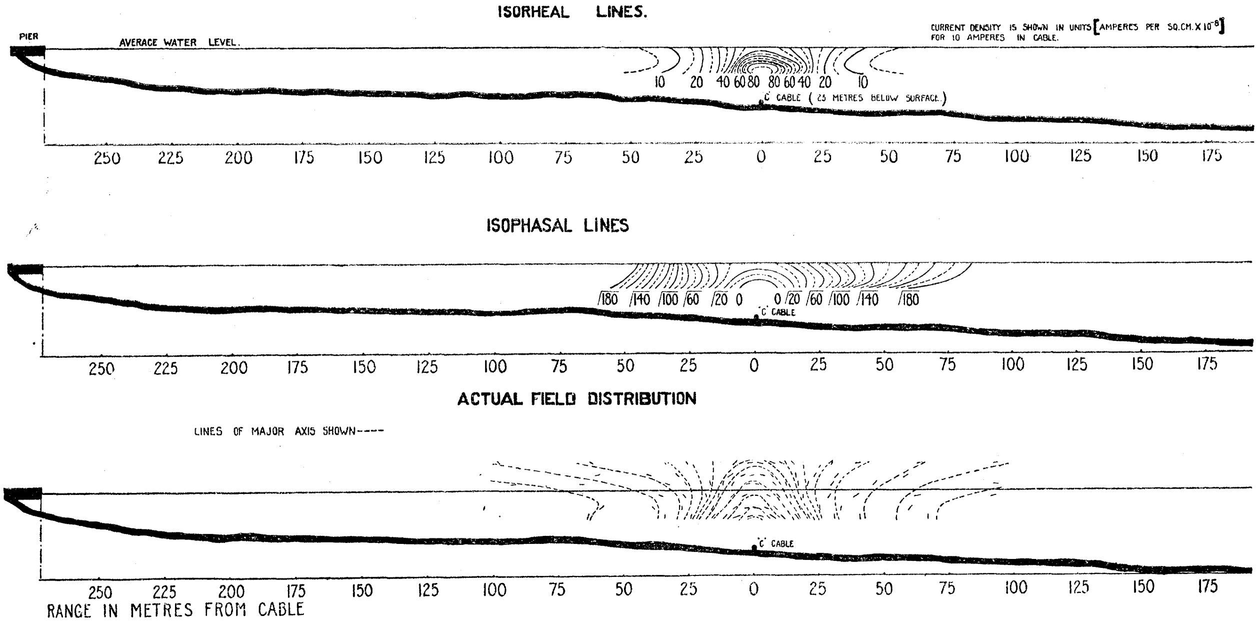

Fig. 22.—Current and Field Distribution around Cable carrying 10 amperes at 500 ∼ per second.

The magnitude and direction of the resultant field has been calculated from the formulæ :—

$$\tan \alpha = \frac{-(H_h^2 - H_v^2) + \{(H_h^2 - H_v^2)^2 + 4H_h^2 H_v^2 \cos^2 \theta\}^{\frac{1}{2}}}{2H_h H_v \cos \theta},$$

$$H_{max.} = [H_h^2 + H_v^2 + \{(H_h^2 - H_v^2)^2 + 4H_h^2 H_v^2 \cos^2 \theta\}^{\frac{1}{2}}]^{\frac{1}{2}} \div \sqrt{2},$$

$$H_{min.} = [H_h^2 + H_v^2 - \{(H_h^2 - H_v^2)^2 + 4H_h^2 H_v^2 \cos^2 \theta\}^{\frac{1}{2}}]^{\frac{1}{2}} \div \sqrt{2},$$

$$\tan \phi = \frac{-(H_h^2 - H_v^2) + \{(H_h^2 - H_v^2)^2 + 4H_h^2 H_v^2 \cos^2 \theta\}^{\frac{1}{2}}}{H_h^2 + H_v^2 + \{(H_h^2 - H_v^2)^2 + 4H_h H_v \cos^2 \theta\}^{\frac{1}{2}}} \tan \theta,$$

where H_h and H_v are the magnitudes of the horizontal and vertical components respectively, θ the angle of phase lag of H_h behind H_v, α the inclination of the major axis, ϕ the angle of lead of $H_{max.}$ in front of H_h.

The following table shows the lateral range in metres from the cable at which the resultant field becomes vertical, for a depth of water of about 75 feet.

Table III.

Frequency per sec.	15 ∼	50 ∼	120 ∼	250 ∼	500 ∼
Lateral range metres	88	56	39	29	20

An interesting feature of the field distribution at 120 ∼ (fig. 20) is the apparent formation of a distinct detached loop on the shore side. This effect has not been found in any other case, and it would appear to be doubtful but for the confirmatory evidence given by a single tilting coil with amplifier and telephones which enabled the direction of the resultant field to be directly determined by inclining the coil to get minimum signals. The directions thus obtained just above the water corresponded very closely with the field distribution deduced from the potentiometer measurements.

A similar loop appeared to be obtained at the same frequency from some measurements taken over the A cable in about 96 feet of water (see fig. 17) but as only one run was made over this cable the evidence is not so conclusive. It is clear that since the wave-length in the water is so short such loops would be formed if it were not for the great attenuation by absorption, but there seems no reason for their only appearing at one frequency. Some evidence of a loop is, however, also shown on the opposite side of the C cable at 250 ~ (see fig. 21).

As a general check on the potentiometer measurements a rectifying amplifier set with microammeter indicator was employed in one set of the observations at 500 ~ and gave fair agreement for the magnitudes of the components. The set was insufficiently sensitive for satisfactory working, owing to the great effect of the cable armouring, which was not known at the time of its design, but the agreement was sufficient to establish confidence in the potentiometer readings.

Distribution of Return Current in the Water.—The measurements of the P.D. between the electrodes enabled the current distribution in the water to be determined with fair accuracy. In order to get a preliminary idea of the average magnitude of the current density in the water a section of the Gareloch was plotted as in Fig. 9 and the area of section determined, from which it appeared that it was $3 \cdot 42 \times 10^8$ square cm. at low water and $3 \cdot 87 \times 10^8$ square cms. at high water, or say, $3 \cdot 65 \times 10^8$ square cms. at mean tide. If, therefore, we consider the earth as non-conducting and an effective current of 10 amperes in the cable, the mean current density in the water should be $27 \cdot 4 \times 10^{-9}$ amperes per square cm. and the average P.D. between the electrodes 10 metres apart in water of 25 ohms per cm.^{3} resistivity would be $0 \cdot 685$ millivolts.

The potentiometer measurements show in accordance with theory that the return current concentrates considerably just above the cable even at the lowest frequency of 15 ~ per sec., and that at mid-depth at a frequency of 500 ~ the current density is over $8{,}000 \times 10^{-9}$ amperes per square cm. In order to exhibit the variations in the magnitude and phase of the return current as clearly as possible lines of equal current density which have been called " isorheal " lines*, and lines of equal phase " isophasals " have been plotted in Figs. 18 to 22 as well as the distribution of the magnetic field deduced from the search coil measurements. Unfortunately time and facilities did not permit of exploring this distribution below about half the depth of the loch, but the results appear to indicate that the current concentrates into quasi-elliptical areas above the cable and is drawn away from the more distant water. For example, with a frequency of 500 ~ the current density at about 80 yards from the cable is only about $2 \cdot 5 \times 10^{-9}$ ampere per square cm.

* From ῥέω, flow, *cf.* rheostat, rheometer.

It will be observed that the isorheal and isophasal lines in the diagrams are in many cases unsymmetrical about the cable. This is probably attributable to the change of depth of water due to the tide during the observations, as each set of readings occupied four or five hours.

The isophasal lines are of great interest if we consider the current in the water as being propagated as a wave from the cable, as they indicate the wave-length and velocity of propagation. The following are the results obtained at the five frequencies.

TABLE IV.—Observed and Calculated Wave-lengths at Various Frequencies.

Frequency ~ per sec.	$\delta\phi$.	$\delta\gamma$ metres.	Wave-length λ in metres.	
			By expt. metres $= 360\,\delta\gamma/\delta\phi$.	By plane wave theory, metres.
15	8°	6·5	293	407·9
50	10	6·25	225	223·6
120	10	4·0	144	144·2
250	10	2·75	99	100
500	20	4·0	72	70·7

It will be seen that at all frequencies except the lowest the agreement between the wave-length determined by experiment and calculated by the plane wave theory for resistivity of 25 per cm.^{3} is remarkably close. No explanation has presented itself for the large discrepancy at 15 ~, as the experimental results appear equally reliable.

Effect of Depth of Water on the Magnetic Field.—In view of the importance of ascertaining the effect of variation of the depth of the water on the intensity of the field above the cable, in connection with piloting ships by the Leader Cable system, a series of separate measurements were made on this point, using the two similar cables which had been laid down in different depths of water. By taking two measurements over each cable at high and low tide respectively the values of the maximum horizontal field for four different depths were determined. Unfortunately the total range of depth thus obtainable was only from about 60 to 105 feet, but this enabled useful results to be obtained. The barge was moored with the coils exactly over the cable at each test, the position being determined by zero E.M.F. on the horizontal coil, while the measurements were made on a vertical coil 10 feet above the surface of the water The following results were obtained :—

TABLE V.—Experimental Measurements of Absorption.

I. *Tests at a frequency of* 50 ~ *per second.*

Effective current = 0·592 ⌈39° × core current.

Depth of water in feet....................	62	72	92	105
Field in microgausses for 10 amperes in cable core	498 ⌈58°	465 ⌈56°	335 ⌈67°	242 ⌈72°
Ditto corrected for armouring H	840 ⌈19°	769 ⌈17°	566 ⌈28°	410 ⌈33°
Calculated field for no absorption H_0	913 ⌈0	802 ⌈0	645 ⌈0	570 ⌈0
Ratio H/H_0	0·920 ⌈19°	0·979 ⌈17°	0·880 ⌈28°	717 ⌈33°

II. *Tests at* 120 ~ *per second.*

Effective current = 0·446 ⌈50° × core current.

Depth of water in feet....................	62	71·5	92·5	105·5
Field for 10 amperes in core	330 ⌈79°	260 ⌈82°	173·5 ⌈106°	138 ⌈105°
Ditto corrected for armouring H	740 ⌈29°	583 ⌈32°	389 ⌈56°	310 ⌈55°
Calculated field H_0	913 ⌈0	807 ⌈0	645 ⌈0	569 ⌈0
Ratio H/H_0	0·810 ⌈29°	0·724 ⌈32°	0·602 ⌈56°	0·544 ⌈55°

III. *Tests at* 250 ~ *per second.*

Effective current = 0·303 ⌈61° × core current.

Depth of water in feet....................	59	72	92·5	102
Field for 10 amperes in core	162·5 ⌈109°	127·5 ⌈121°	62·8 ⌈144°	49·5 ⌈153°
Ditto corrected for armouring H	536 ⌈48°	421 ⌈60°	207 ⌈83°	163·5 ⌈92°
Calculated field H	953 ⌈0	802 ⌈0	640 ⌈0	587 ⌈0
Ratio H/H_0	0·562 ⌈48°	0·525 ⌈60°	0·323 ⌈83°	0·279 ⌈92°

IV. *Tests at* 500 ~ *per second.*

Effective current = 0·173 ⌈75·5° × core current.

Depth of water in feet....................	66·3	70·8	93	102
Field for 10 amperes in core	39·2 ⌈129°	33·0 ⌈135°	19·2 ⌈216°	18·0 ⌈231°
Ditto corrected for armouring H	227 ⌈52·5°	192 ⌈58·5°	111 ⌈139·5°	104 ⌈156·5°
Calculated field H_0	860 ⌈0	814 ⌈0	638 ⌈0	588 ⌈0
Ratio H/H_0	0·264 ⌈52·5°	0·236 ⌈58·5°	0·174 ⌈139·5°	0·177 ⌈156·3°

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The curves, figs. 23 and 24, show the variation of the intensity and of phase of the field with depth, and by plotting log H/H_0 against depth we obtain fairly straight

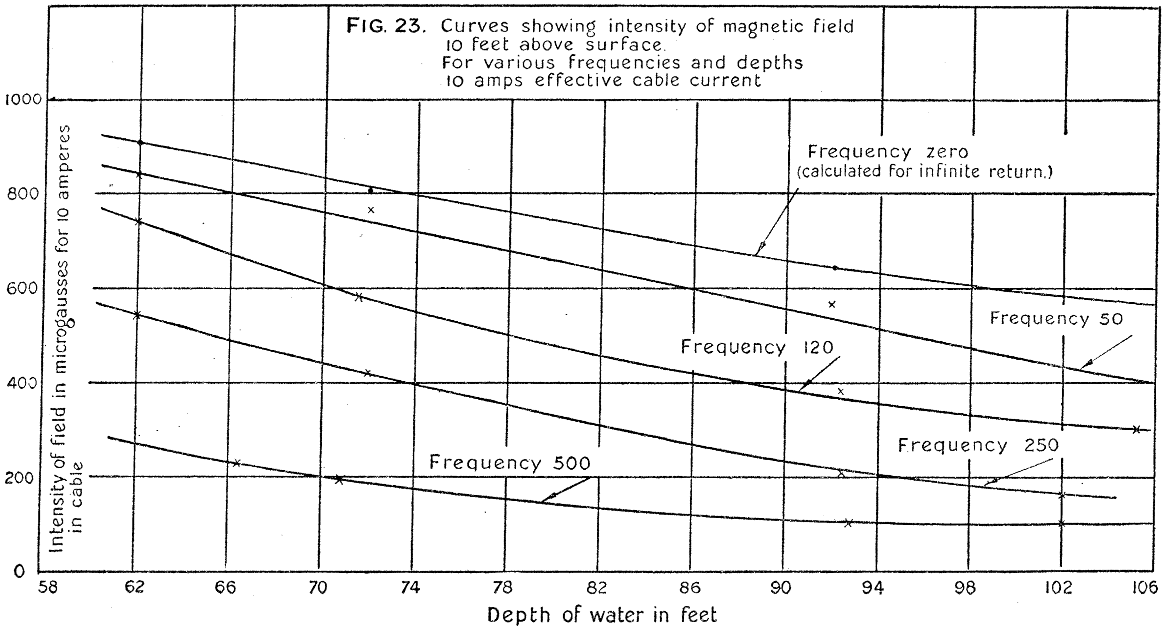

FIG. 23. Curves showing intensity of magnetic field 10 feet above surface. For various frequencies and depths 10 amps effective cable current

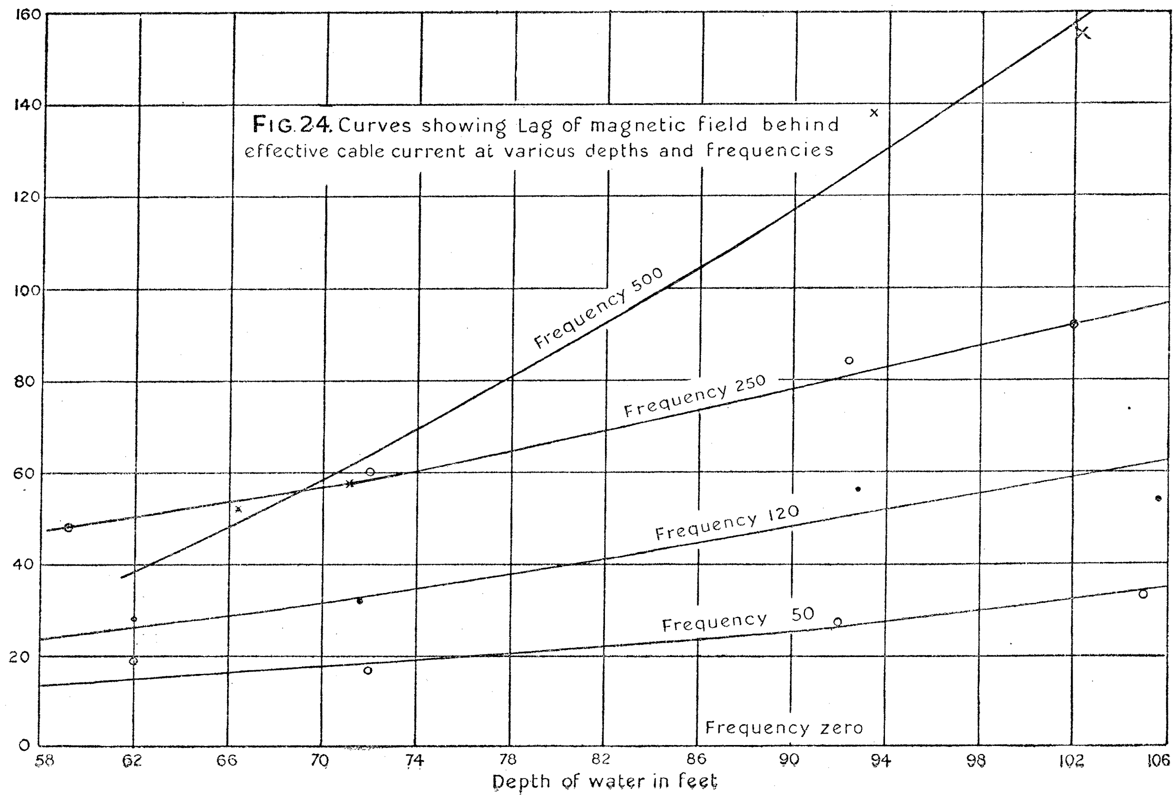

FIG. 24. Curves showing Lag of magnetic field behind effective cable current at various depths and frequencies

lines indicating an exponential law of absorption for each 10 metres of depth. The following table shows the ratio of diminution by absorption at various frequencies, compared with the values calculated by Mr. BUTTERWORTH from his theory, and those calculated on the ordinary plane-wave theory. The resistivity of the water in these experiments was about 35 ohms per cm.³ owing to cold weather, and the calculations have been made on that basis.

TABLE VI.—Experimental and Theoretical Values of Absorption.

Frequency ~ per second.	50 ~	120 ~	250 ~	500 ~
Ratio of diminution by absorption for each 10 metres of depth—				
(a) by experiment	0·848	0·754	0·585	0·617
(b) by Mr. Butterworth's theory ...	0·853	0·769	0·665	0·535
(c) for plane waves	0·790	0·694	0·588	0·472
Phase lag for each 10 metres—				
(a) By experiment	14·2°	27·5°	36·0°	106°
(b) by Mr. Butterworth's theory ...	11°	19°	28°	42°
(c) for plane waves	13·6°	21·1°	30·5°	43°

Here the agreement between the experimental results and those obtained by theory is very fair at the lower frequencies but not at 500 ~. These 500 ~ results were only obtained during the last few days before the station closed, and there was no opportunity for thoroughly examining and eliminating disturbances.

This rate of diminution with depth is much greater than that found by H. LICHTE at Kiel, but it would appear that he worked in the cold water of the Baltic, so that the resistivity must have been much higher, and in fact he gives it as having been 100 ohms per cm.³ But the agreement between theory and experiment is sufficiently good to allow of the former being used with confidence.

By the principle of similitude mentioned on p. 98, the results for a given depth and frequency should be the same as for another depth and frequency provided $d\sqrt{f}$ is the same in both cases. In fig. 25 the whole of the values of H/H_0 and of the phase lag ϕ have been plotted against $d\sqrt{f}$ and the majority of the observations lie fairly well on the two lines.

Effect of Cable Armouring.—Reference has already been made to the concentration of the return current in the armouring owing to its superior conductivity and to the skin effect, so that the effective current in an armoured cable may be much less than the core current. This is a difficult matter to determine experimentally, as it must be done on a considerable length of submerged cable and provision must be made for

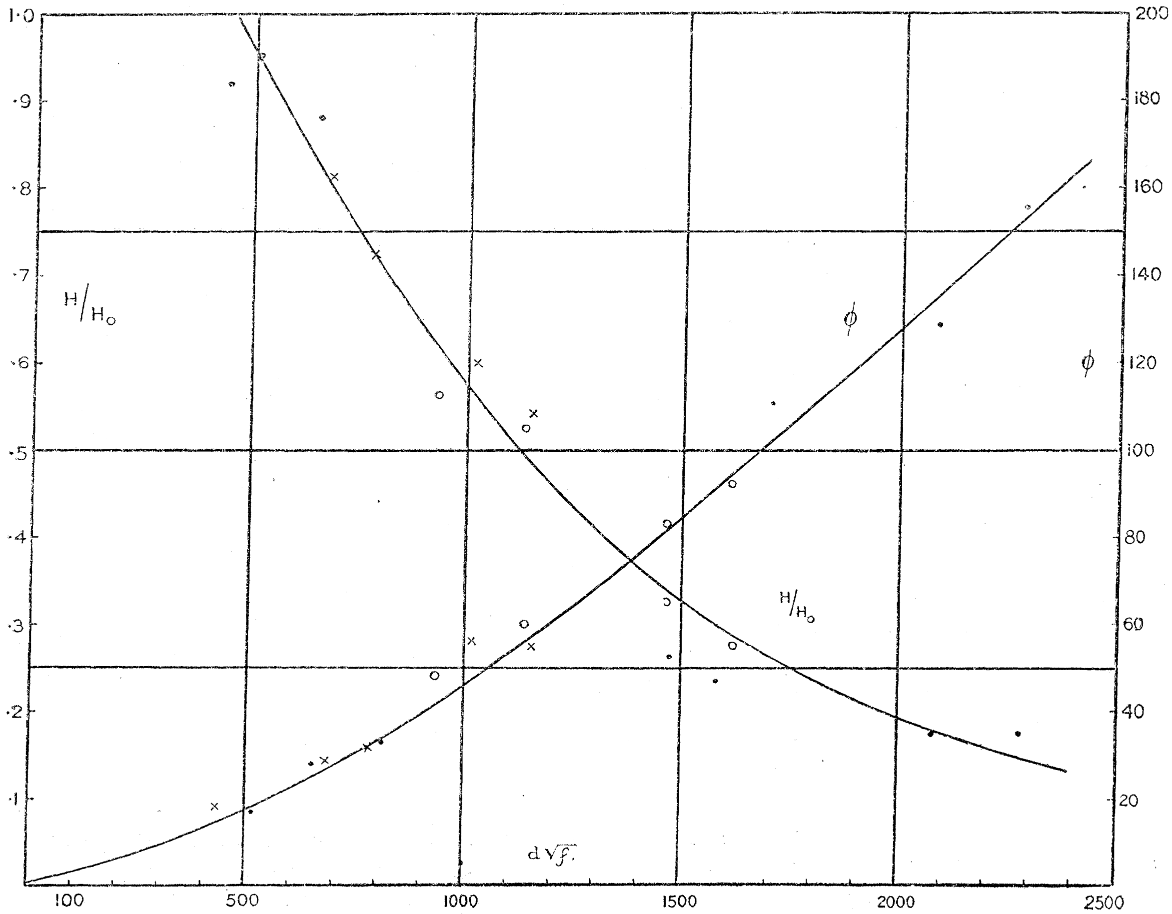

FIG 25. Variation of the ratio of Actual Field to undisturbed Field.

measuring both the effective and core currents preferably near the centre of the run. As the arrangements of the Gareloch cables permitted the magnitude and phase of the core current to be measured, arrangements were made to determine the effective current by means of a special ring-shaped transformer with hinged core which could be clipped over the cable by a diver. By the kindness of the R.N. Torpedo Factory, Greenock, this transformer was fitted on the cable in March, 1922, careful measurements having first been made of the magnitude and phase of the secondary E.M.F. when clipped on a conductor carrying a known current at various frequencies. The leads from the transformer when clipped on the cable were brought up to a terminal box on a dan buoy, and the barge carrying the potentiometer was moored over the cable and successive readings taken of the magnitude and phase of the core current and of the E.M.F. in the transformer secondary.

A mathematical theory of the return current in the armouring has been given by O. Oldenburg* and Mr. Butterworth has reduced his result to the form

$$I_{\text{eff}} = I/(1+jx).$$

Where I is the core current, I_{eff} the effective current,

$$x = \frac{8\pi^2 \rho r t}{\rho' L^2}\left\{\log_e \frac{L}{7\cdot 91 r} + j\frac{\pi}{4}\right\},$$

ρ = resistivity of the water, ρ' resistivity of the armouring, $L = \sqrt{\rho/f}$ the wave-length in water at frequency f, t = thickness of armouring, and r overall radius of cable.

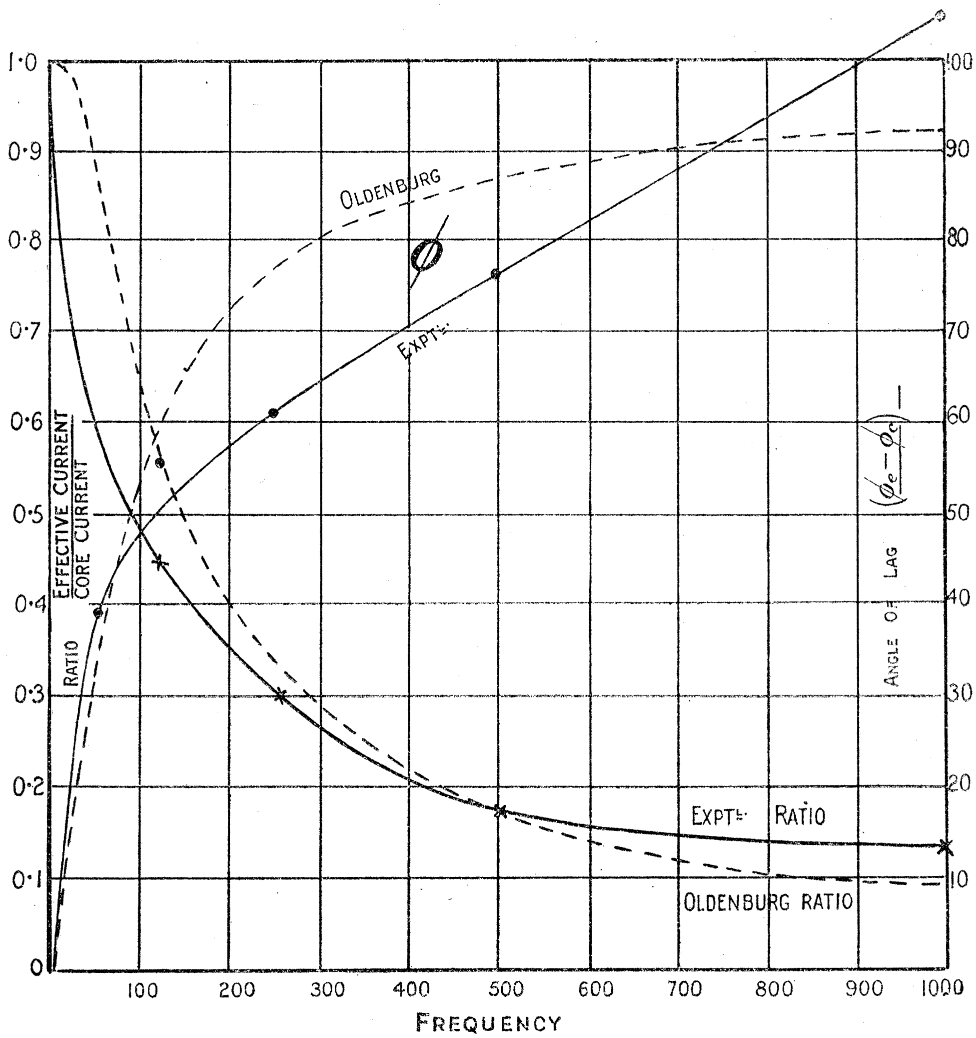

Fig. 26.—Magnitude and Phase of Effective Current in Armoured Submarine Cable.

Taking $\rho = 3 \times 10^{10}$, $\rho' = 12{,}000$, $t = 0\cdot 2$ cm. and $r = 1\cdot 27$ cm. for the cable

* 'Archiv für Elektrotechnik,' IX, vol. 7, p. 289 (1920).

employed, Mr. BUTTERWORTH has calculated the magnitude and phase of the effective current, and the following table shows the observed and calculated results :—

TABLE VII.—Experimental and Theoretical Effect of Cable Armouring.

Frequency per sec.	Core Current I Amps.	Transformer E.M.F. E Volts.	Effective Current $I_{eff} = \frac{2080\ E}{\text{Frequency}}$.	Ratio I_{eff}/I.	
				Observed.	Calculated.
			Amps.		
50	10·82	$0\cdot154\ \underline{\mid 51^\circ}$	6·41	$0\cdot592\ \overline{\mid 39^\circ}$	$0\cdot89\ \overline{\mid 34^\circ}$
120	9·04	$0\cdot232\ \underline{\mid 34^\circ}$	4·03	$0\cdot446\ \overline{\mid 56^\circ}$	$0\cdot60\ \overline{\mid 60^\circ}$
250	5·57	$0\cdot202\ \underline{\mid 29^\circ}$	1·68	$0\cdot302\ \overline{\mid 61^\circ}$	$0\cdot335\ \overline{\mid 76\cdot5^\circ}$
500	6·99	$0\cdot288\ \underline{\mid 13\cdot5^\circ}$	1·21	$0\cdot173\ \overline{\mid 76\cdot5^\circ}$	$0\cdot181\ \overline{\mid 82^\circ}$
1,000	2·19	$0\cdot138\ \overline{\mid 14\cdot5^\circ}$	0·287	$0\cdot131\ \overline{\mid 104\cdot5^\circ}$	$0\cdot090\ \overline{\mid 92^\circ}$

Fig. 26 shows the comparison between the theoretical and observed values, and the serious diminution of the effective current in even this lightly armoured cable at the higher frequencies such as are used in ordinary Leader Gear working. There is a considerable discrepancy between theory and experiment at the lower frequencies, and the experimental results have been employed in correcting the field and current measurements, as there seemed no reason to doubt their accuracy. The comparison with Mr. BUTTERWORTH'S calculations of field intensity, however, casts some doubt on the low frequency experimental results, as before mentioned.

CONCLUSIONS.

The following conclusions appear to be justified by the results of the theoretical and experimental observations :—

(*a*) The propagation of plane electromagnetic waves in sea water appears to take place in close accordance with the theory of wave propagation in a conducting medium, the wave-length λ being equal to $\sqrt{\rho/f}$, the velocity of propagation $v = \sqrt{\rho f}$, and the attenuation constant $a = 2\pi\sqrt{f/\rho}$. At low frequencies the velocity of propagation of electromagnetic waves is only of the same order as that of acoustic waves.

(*b*) Owing to the "skin effect" in the conducting water the return current tends to concentrate near to the cable and above it if the bottom is relatively non-conducting. The higher the frequency the greater the concentration, and the effect appears to be that of a diffused mirror image of elliptical form above the cable, causing the magnetic field distribution to approximate more and more nearly to that produced by a pair of leading and return conductors in a vertical plane as the frequency is increased.

(*c*) The variation of phase of the return current in the water above the cable appears both by theory and experiment to correspond fairly closely with the propagation of plane electromagnetic waves in an infinite sea.

(*d*) The fundamental equations for wave propagation in a conducting medium of any form lead to a principle of similitude by which the whole of the phenomena can be reproduced in another medium of similar form if the scale is altered in the ratio of $\sqrt{\rho/\mu f}$ or of the wave-lengths for the two mediums. This enables the field distribution at large distances from the cable to be determined by employing a metallic sheet in place of the water.

(*e*) At high frequencies theory indicates (and the experimental results appear to be in conformity with it) that the magnetic line of force above and below water are in the form of circles tangential to the surface at points vertically above and below the cable. This is explainable by the fact that since the velocity of propagation in the air is of the order of a million times greater than in sea water the lines of force which would travel outwards in circles concentric with the cable in the absence of water are retarded so greatly in the conducting lamina as to be relatively stationary so that they are like expanding rings pulled into the centre equatorially.

(*f*) The magnetic field at the surface vertically above the cable diminishes with increase of depth both due to increase of distance from the cable and to absorption. The latter appears to follow an approximately exponential law agreeing fairly closely with theory.

(*g*) In the region where the phase of the horizontal component of the magnetic field rapidly changes, the field is of an elliptically rotating character, and the greatest ratio of minor to major axis observed has been about 0·5 at 500 ∼. It appears probable that at higher frequencies pure rotating fields may be obtained. Immediately above the cable and at large lateral ranges the field is almost purely alternating.

(*h*) Armouring a submarine cable produces a marked diminution of the magnetic field, and of the return current in the water owing to the concentration of the return current in the sheath. The effective or resultant current found by experiment is in fair agreement with the theory of OLDENBURG at frequencies in the neighbourhood of 500 ∼, but is lower than the theoretical value at low frequencies.

(*i*) The experiments on a steel ship give a distribution of the lines of force above the surface which differs from that obtained on a wooden vessel mainly in that the lines of force are horizontal in the case of a steel ship where they would ordinarily be vertical in its absence. It is obvious that the superior magnetic permeability of the steel ship cannot account for this phenomenon, and the explanation apparently lies in the greater electrical conductivity of the hull as compared with the water which causes a considerable concentration of the return current in it, thus producing a magnetic field encircling the hull and superposing a somewhat strong horizontal component above the deck. Calculations made by Mr. BUTTERWORTH on this point support *this hypothesis*.

The experimental investigations above described together with Mr. BUTTERWORTH'S theoretical calculations appear to afford a fairly complete solution of the problem undertaken, and data have now been obtained which will enable the complete design of Leader Gear installations to be undertaken, and which should be of value in any question regarding underwater signalling by electromagnetic impulses. The writer's thanks are cordially tendered to his assistants, Mr. S. J. WILLIS and Mr. L. CHAMPNEY, B.Sc., who had the difficult task of equipping the Gareloch Sub-station and testing barge and of making the majority of the observations and calculations. He is also indebted to Mr. J. H. POWELL, M.Sc., and Mr. J. A. CRAIG for much help in the earlier observations, and especially to Mr. S. BUTTERWORTH, M.Sc., for his mathematical investigations, which are being published separately. Thanks are also tendered to the Physics Board of the Department of Scientific and Industrial Research for the grant which enabled the investigation to be carried on after the closing down of the Admiralty Experimental Station at Shandon.

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